STATISTICAL ASPECTS OF FOOD QUALITY ASSURANCE

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STATISTICAL ASPECTS OF FOOD QUALITY ASSURANCE

Subhash C. Puri
Chief Statistician
Food Production & Inspection Branch
Agriculture Canada, Ottawa

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PREFACE

Quality assurance has come to play a very significant role in all aspects of industrial and nonindustrial production processes. At the same time, the application of statistical methodology is indispensable to the development of any quality assurance program.

This book is intended to provide important statistical aspects of quality assurance essential to its development, understanding, and implementation by all concerned parties, viz., producer, consumer, and regulatory agency. It is a working manual and is especially written for personnel of Agriculture Canada engaged in quality assurance of food and agricultural products.

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S.C. Puri

To Pamela & Anuradha

CHAPTER 1
INTRODUCTION

1.1 TOTAL QUALITY PROGRAM

The principles and techniques of quality control have come to play an increasingly significant role in the development of efficient food, agricultural, and health systems. Quality programs are being generated for each activity. There is a general desire to quantify all information as much as possible and analyze it scientifically, to enable us to appraise it objectively. A total quality program is a collection of all activities and events that are provided to ensure that a product, process, or service will satisfy given needs.

There are generally three principal parties involved in a contract to produce goods: the consumer, the producer, and the acceptance/regulatory authority. The consumer specifies the quality requirements, the producer undertakes to meet them, and the regulatory authority confirms that the quality requirements have been met. It is through the collective effort and satisfaction of all three parties that the ultimate goal is achieved, that is, production of a high-quality product that is satisfactory, economical, and reliable.

In a regulatory agency such as ours, the objective is to assist the producer realize a fair return for his management, capital, and labor, and at the same time insure the consumer gets a high-quality product that meets established standards and specifications. To achieve this objective, we need a comprehensive and integrated quality assurance system. Essentially, for our purposes, such a system requires the development of:

— proper specifications/tolerances/regulations;
— a statistically sound sampling inspection system; and
— an evaluation and review/revision procedure.

1.2 DEFINITIONS AND TERMINOLOGY

1. *Quality* can be defined as the sum of all characteristics that determine the acceptance of a product in relation to the intent of design, specifications, and consumer expectations.

2. *Quality control* refers to the control of all activities encountered in the process of producing or regulating a satisfactory, reliable, and economical product.

3. *Statistical quality control* refers to the application of statistical techniques to quality problems for the scientific analysis and interpretation of data.

4. *Quality assurance* refers to the assurance that a customer will continuously receive a product that satisfies all the requirement embodied in the intent of production.

5. *Quality reliability* refers to the chance or probability that the marketed product will function without problems or failure in accordance with the design, specification, and the required duration.

1.3 PLAN OF THE BOOK

Besides the various management aspects of production, development, and marketing in any total quality program, there are some fundamental statistical aspects that play a significant role in the understanding and development of quality systems. These involve setting up a sampling inspection system (i.e. how much to sample, how to sample and how often to sample) and studying product/process variability. The book explains methods and techniques to satisfy these fundamental needs and, therefore, includes:

— basic statistical concepts;
— sample selection methods;
— acceptance sampling; and
— control charts.

Though the book has been written in the context of activities and functions associated with a regulatory agency, the statistical methods given can be universally applied to any other situation. The more ambitious reader will find a selected bibliography at the end.

CHAPTER 2
BASIC STATISTICAL CONCEPTS

2.1 INTRODUCTION

This chapter is concerned with the evaluation of measurements by means of statistical methods. From the statistical viewpoint, each measurement or a group of measurements (sample) is considered as only one realization of a hypothetical infinite population of similar measurements. Although, in general, all members of this population refer to the measurements of the same property on the same sample, they are not expected to be identical. The differences among them are attributable to chance factors as well as multitude of other assignable factors associated with the measuring process. Our aim is, therefore, to identify these causes of variation, evaluate their significance, and ultimately find means to make inferences from a sample to a population.

A *population* is the collection of all individuals on which one or more variables are to be observed. A population can be finite or infinite, real or hypothetical. We may speak of the population of human beings in Canada, number of oranges in an orchard, or number of grains of sand in the world. Most populations can be characterized in terms of very few quantities, called parameters. Two important ones are the *mean* and the *standard deviation*, measuring respectively, the location of the center of the population and its spread.

A *sample* is a portion of the population. An important type of sample in statistics is a *random sample*. This is one that has been selected by a random process and is such that each measurement in the population has an equal and independent chance of being included in the sample. Whereas a population is described by its parameters, a sample is described by its *statistics* or *sample statistics*.

The numerical data yielded by an experiment or process is of two types: *discrete* or *continuous*. In discrete data, the variable can assume only specific values (usually integers) and involves counting, for example, number of cows on a farm. In continuous data, the results are measured, for example, temperature readings. In a process yielding discrete data, the relevant quantity is often called a *attribute*; in a process yielding continuous data, the relevant quantity is often called a *variable*.

2.2 FREQUENCY DISTRIBUTION AND HISTOGRAM

Statistical data from a scientific study usually consists of a large number of observations. To obtain meaningful information, this unorganized set of values has to be concisely summarized, described, and presented. The usual visual technique for presenting such data is by a *histogram* or *bar graph*. For discrete data, these graphs are generally not difficult to construct. Continuous data, such as weight, temperature, pressure, length, and percentage are not already grouped into natural categories, and must first be arranged into some convenient

grouping. This is done with a *frequency table*, whereby the range of the data is divided into a moderate number of categories and each observation is placed in one of the categories. The number of categories is arbitrary, but a good rule of thumb is to let k = n, where k is the number of categories and n is the number of observations. If R is the range of the data (range = largest observation minus the smallest observation), then the width of each category is approximately R/k. For simplicity, we will only consider situations in which each category has the same width.

EXAMPLE: Table 2.1 gives moisture content (%) of skim milk powder obtained through laboratory analysis of 60 samples.

TABLE 2.1 Percent moisture in skim milk powder

3.3	3.6	3.6	3.5	2.9	3.5	3.8	3.3	3.5	2.8
3.4	3.4	3.7	3.6	3.1	2.9	4.0	3.5	3.5	3.3
3.5	2.6	3.5	3.9	3.2	3.6	3.4	3.2	3.6	2.5
3.1	2.9	3.2	4.0	3.8	3.5	2.8	3.5	3.5	3.4
3.2	3.4	3.0	2.8	3.9	3.5	2.7	4.0	3.5	2.6
3.5	3.0	3.4	3.6	3.0	3.2	2.9	3.0	2.7	3.4

The number of categories should be approximately $\sqrt{60}$, or k = 8. The range of the data, 4.0 — 2.5 is 1.5. The consecutive categories, each of width 0.2, are 2.5 — 2.7, 2.7 — 2.9, . . . , 3.9 — 4.1. As a convention, any observation falling on the border of two categories will be put in the upper of the two. The frequency table may be presented as in Table 2.2.

TABLE 2.2 Frequency table for moisture (%) in skim milk powder

Class boundaries	Class midpoint (X)	Frequency (f)	Cumulative frequency
2.5 — 2.7	2.6	3	3
2.7 — 2.9	2.8	5	8
2.9 — 3.1	3.0	8	16
3.1 — 3.3	3.2	7	23
3.3 — 3.5	3.4	11	34
3.5 — 3.7	3.6	18	52
3.7 — 3.9	3.8	3	55
3.9 — 4.1	4.0	5	60

Once the data have been arranged into a frequency table, they may be presented in a histogram (see Figure 2.1) by plotting the limits (on the horizontal axis) against the frequency (on the vertical axis). One assumes that all observations in the same category have the category midpoint as their value.

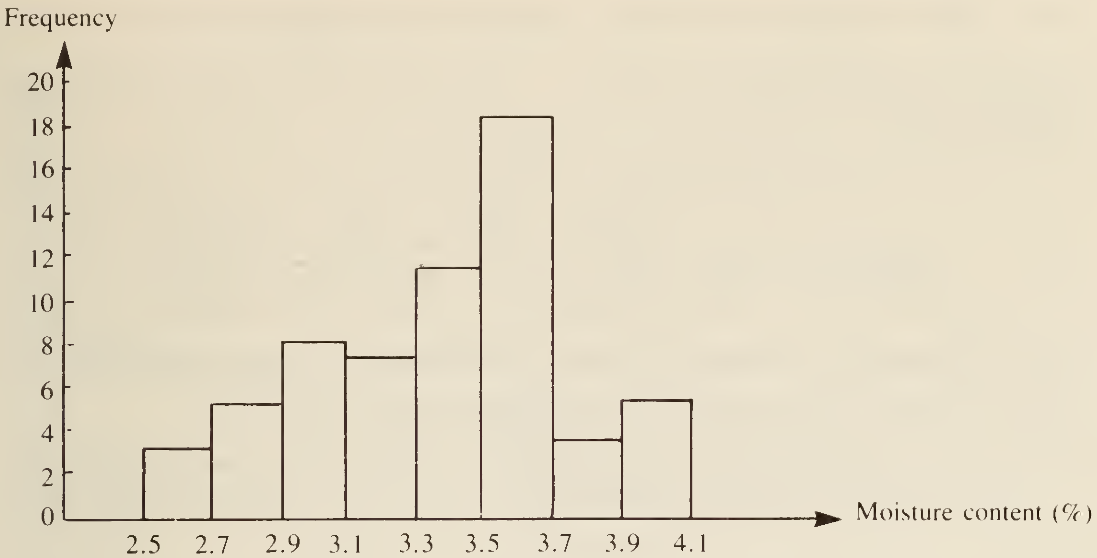


FIGURE 2.1 Histogram for moisture content (%) of skim milk powder

2.3 STATISTICAL MEASURES

2.3.1 The mean

For a sample of observations $X_1, X_2, \ldots, X_n$, the sample *arithmetic mean* (or *sample mean* or, simply, the *mean*), denoted by $\overline{X}$ is

$$\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$$

The corresponding *population mean* is denoted by the Greek letter μ.

For grouped data where the variables $X_1, X_2, \ldots, X_n$, occur with respective frequencies $f_1, f_2, \ldots, f_n$, the mean is defined as

$$\overline{X} = \frac{X_1 f_1 + X_2 f_2 + \ldots + X_n f_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} X_i f_i}{\sum_{i=1}^{n} f_i}$$

2.3.2 The range

The range for a set of observations is the difference between the largest and smallest observations. For a frequency table, the range is the difference between the upper boundary of the highest class and the lower boundary of the lowest class.

2.3.3 The variance and the standard deviation

For a set of n observations, $X_1, X_2, \ldots, X_n$, the *sample variance* denoted by S^2 is defined as

$$\text{Variance} = S^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n - 1} = \frac{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}{n - 1}$$

The corresponding *population variance* is denoted by σ^2 (sigma squared).

The square root of the variance is called the *standard deviation*.

In case of a grouped frequency distribution, where a set of numbers $X_1, X_2, \ldots, X_n$ have their respective frequencies $f_1, f_2, \ldots, f_n$, the variance is given by

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2 f_i}{\sum_{i=1}^{n} f_i - 1} = \frac{\sum X_i^2 f_i - \frac{(\sum X_i f_i)^2}{\sum f_i}}{\sum f_i - 1}$$

EXAMPLE: Using the data of Table 2.1, find the mean, variance and standard deviation of moisture content (%) for the 60 observations given.

For the raw data,

$$\overline{X} = \frac{3.4 + 2.7 + \ldots + 3.3}{60} = \frac{199.2}{60} = 3.32$$

$$\sum X_i^2 = 669.52, \ \sum X_i = 199.2$$

$$S^2 = \frac{669.52 - \frac{(199.2)^2}{60}}{60 - 1} = 0.1386$$

$$S = \sqrt{0.1386} = 0.3723$$

For the grouped data,

$$\overline{X} = \frac{(3)\,(2.6) + (5)\,(2.8) + \ldots + (5)\,(4.0)}{60} = \frac{201.8}{60} = 3.36$$

$$\sum X_i^2 f_i = 686.92, \ \sum X_i f_i = 201.8$$

$$S^2 = \frac{686.92 - \frac{(201.8)^2}{60}}{60 - 1} = 0.1390$$

$$S = \sqrt{0.1390} = 0.3728$$

2.4 PROBABILITY

There are two approaches to defining probability: *classical*, and as a *relative frequency*. According to classical approach, if a procedure gives rise to n equally likely outcomes, of which r have attribute A, then the probability of A is r/n. This definition is somewhat limited because to calculate any probabilities we need to know the value of n and be sure that each outcome is equally likely. Since most food and agricultural problems do not satisfy these requirements, a more pragmatic definition of probability, called the frequency concept of probability, is used. We shall interpret probability as a relative frequency in a large number of trials, i.e. when we talk of the probability of an event A, we mean the relative frequency of A in a large number of similar trials. If an event of interest, A, occurs r times in n trials, and if the ratio r/n approaches a limit as n becomes large, then r/n is called the probability of A, and we write $P(A) = r/n$.

One sees from either definition of probability that $P(A) \geq 0$, $P(A) \leq 1$, and if $\bar{A}$ represents the complement of A, then $P(\bar{A}) = 1 - P(A)$. Note that, if an event A is impossible, then $r = 0$ and $P(A) = 0$. If an event A is certain (i.e. it occurs at every trial), then $r = n$ and $P(A) = 1$.

If, for example, a sample of 200 oranges is inspected from a large consignment, and 10 are found to be diseased, then the proportion of diseased oranges (or the relative frequency of diseased oranges) in the consignment is approximately 10/200. Thus, the probability that an orange is diseased [written P (an orange is diseased)] is 0.05.

2.5 PERMUTATIONS AND COMBINATIONS

For a set of n objects, an arrangement of r of them in a definite order is called a *permutation*. The number of different permutations is denoted by n_{P_r} and calculated by the following formula

$$n_{P_r} = \frac{n!}{(n - r)!}$$

Where $n! = n(n - 1)(n - 2) \ldots 1$, and $(n - r)! = (n - r)(n - r - 1) \ldots 1$, and $0!$ is taken to be 1. For example, $3! = 3.2.1 = 6$.

For a set of n objects, a subset of r of them (chosen without regard to their order of selection) is called a *combination*. The number of different combinations is denoted by $\binom{n}{r}$ and calculated by the following formula

$$\binom{n}{r} = \frac{n!}{r!\,(n - r)!}$$

If we consider n repeated trials, each with two possible outcomes (e.g. defective or not defective), then the possible number of different arrangements each having X defectives and $n - X$ nondefective is $\binom{n}{X}$.

2.6 FOUR IMPORTANT DISTRIBUTIONS

There are three important distributions that deal with discrete data: *binomial*, *Poisson*, and *hypergeometric*. For measured data, called continuous data, probabilities are calculated using the *normal distribution*.

2.6.1 Binomial distribution

If p is the probability that any object is defective, then the probability that in a random sample of n objects there will be X defective is given by the binomial distribution whose formula is given below.

$$P\ (X \text{ defective among n objects}) = \binom{n}{X} p^X (1-p)^{n-X}$$

EXAMPLE: A labelling process is known to produce 20% defective items. What is the probability of finding two defective in a sample of four? Here, $n = 4$, $X = 2$, $p = 0.2$.

$$\begin{aligned} P\ (2 \text{ defective}) &= \binom{4}{2} (0.2)^2 (0.8)^2 \\ &= 0.1536 \end{aligned}$$

2.6.2 Poisson distribution

In case of a rare, random event, where n is large and p is small, the probabilities are calculated using Poisson distribution as follows:

$$P\ (X \text{ defective in n objects}) = \frac{e^{-\lambda} \lambda^X}{x!}$$

where $\lambda = np$, and $e = 2.718$. Approximation of binomial probabilities, using the Poisson distribution, is generally adequate if n is larger than 20 and p is smaller than 0.05. If n is larger than 100, then p may be as large as 0.1.

EXAMPLE: Reconsider the above example of the labelling process. Suppose that the proportion of defectives is 5%, and every hour a sample of 40 items is taken. What is the probability of finding 1 defective item.

Here, n is large (40), p is small (0.05), and $\lambda = np = 2$. Using Poisson distribution,

$$P\ (1 \text{ defective}) = \frac{e^{-2}\, 2^1}{1!} = 0.2707$$

2.6.3 Hypergeometric distribution

To calculate probabilities of samples from small populations, one uses the hypergeometric distribution. If the population contains N units of which X are defective, and a sample of n units is randomly selected, the probability of finding x defective units in the sample is given by

$$P\ (x) = \frac{\binom{X}{x}\binom{N-X}{n-x}}{\binom{N}{n}}$$

EXAMPLE: If the population consists of 20 objects, of which two are defective, and a sample of five objects is examined, the probability of its containing one defective object is

$$\frac{\binom{2}{1}\binom{18}{4}}{\binom{20}{5}} = 0.3947$$

2.6.4 Normal distribution

The most important distribution for dealing with continuous or measured data is called the normal distribution, whose formula is

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-1/2\left(\frac{X-\mu}{\sigma}\right)^2} \text{ for all values of X,}$$

where μ and σ are the mean and standard deviation of the population. Figure 2.2 shows the graph of a normal distribution. The probability that a variable X, which has a normal distribution with mean μ and standard deviation σ, lies between two values a and b is the area under the distribution curve between a and b.

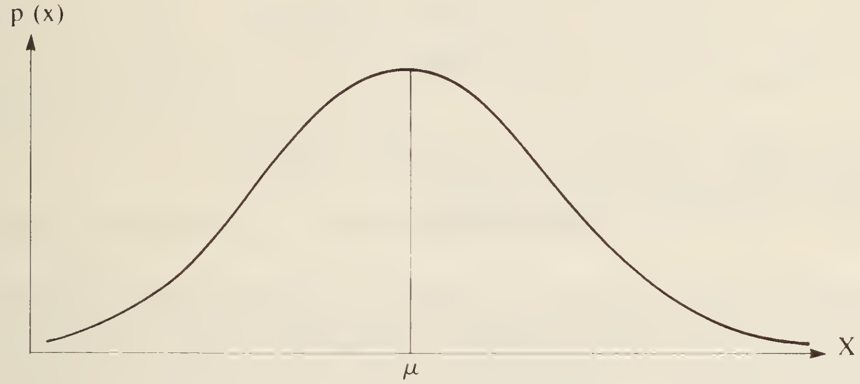


FIGURE 2.2 A typical normal distribution

Some important features of the normal curve are:

1. Once μ and σ are specified, the normal curve is completely determined.
2. The curve is symmetrical about a vertical axis through the mean. The observations tend to cluster at the mean.
3. The total area under the curve is equal to 1.
4. Although the curve extends to infinity on both sides, for all practical purposes there is negligible area beyond 3σ on each side. Empirically, the following is known:

   68.26% of area is encompassed between $\mu \pm 1\sigma$
   95.44% of area is encompassed between $\mu \pm 2\sigma$
   99.74% of area is encompassed between $\mu \pm 3\sigma$

The normal curve, being dependent on μ and σ, changes shape with different values of μ and σ, thereby, generating a large family of distributions. It would

be a hopeless task to set up separate tables of normal probabilities and areas for every conceivable value of μ and σ. Fortunately, it is not necessary to do so. We transform the normally distributed random variable X with mean μ and standard deviation σ to a new random variable, called Z, as follows:

$$Z = \frac{X - \mu}{\sigma}$$

which has a *standard normal distribution* with $\mu = 0$ and $\sigma = 1$. The curve of the standard normal distribution is centered at zero and has the bulk of its area between -3 and 3. The probability that Z lies between any two numbers a and b (assuming that $a < b$) is read from Appendix Table 1.

EXAMPLE: Using the standard normal distribution of Appendix Table 1, find the probability that Z is between -2 and 1.

The area between -2 and 1 is the sum of the areas between -2 and 0 and 0 and 1. Also, by symmetry, the area between -2 and 0 is the same as between 0 and 2. Thus reading the two areas from Appendix Table 1, we have,

$$\begin{aligned} p(-2 < Z < 1) &= p(0 < Z < 1) + p(0 < Z < 2) \\ &= 0.3413 + 0.4773 \\ &= 0.8186 \end{aligned}$$

2.7 DISTRIBUTION OF SAMPLE MEAN AND SAMPLE VARIANCE

If the population has N elements in it, then the possible number of samples of size n is $\binom{N}{n}$, and for each of these possible samples there is a mean $\overline{X}$. These $\overline{X}$'s have a distribution of their own. If the original population has a normal distribution, then $\overline{X}$ itself also has a normal distribution. If the original population is not normal, then a very important theorem in mathematical statistics, called the *central limit theorem* tells us that the distribution of $\overline{X}$ is approximately normal, and that the approximation improves as n gets bigger. If μ and σ are the mean and standards deviation of the original population, then the mean and standard deviation of the distribution of $\overline{X}$, called $\mu_{\overline{X}}$ and $\sigma_{\overline{X}}$ respectively, are

$$\mu_{\overline{X}} = \mu, \ \sigma_{\overline{X}} = \sigma \sqrt{\frac{N - n}{n(N - 1)}}$$

For large populations (N large), $\sigma_{\overline{X}}$ is approximately equal to $\sigma/\sqrt{n}$, $\sigma_{\overline{X}}$ is known as the *standard error*. Note that the standard deviation refers to the variation of the observations on individual units, whereas the standard error refers to the random variations of an estimate from a whole experiment. If a variable has any normal distribution, it may be reduced to a standard normal distribution by subtracting the mean and dividing by the standard deviation. Since the distribution of $\overline{X}$ is normal with mean μ and standard deviation $\sigma/\sqrt{n}$, one can reduce it by subtracting μ and dividing by $\sigma/\sqrt{n}$. Thus we have

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

2.8 t-DISTRIBUTION

A basic difficulty is that σ is generally unknown and has to be estimated by the sample standard deviation S. Then, subtracting μ and dividing by $S/\sqrt{n}$ no longer has a standard normal distribution, but a *t-distribution*. The t-distribution is symmetric, bell-shaped, and centered at zero, just as the standard normal distribution is. However, there is not a single t-distribution but a family of them, each member of the family being distinguished by its *degrees of freedom*, f, defined as $f = n - 1$.

Whereas, in the standard normal distribution, 95% of the area lies between ± 1.96 (from Appendix Table 1), in the t-distribution, there is a number, which we call $t_{.975}$, so that 95% of the area lies between $-t_{.975}$ and $+t_{.975}$. The actual values of $t_{.975}$ change for different degrees of freedom, and are given in Appendix Table 2.

EXAMPLE: Using Appendix Table 2, find the two numbers on the t-distribution which contain 95% of the area of the distribution for $n = 15$.

For $n = 15$, $f = 14$, $t_{.975} = 2.1448$. Thus 95% of the t-distribution lies between -2.1448 and 2.1448. Notice that as n increases, the values of $t_{.975}$ and $-t_{.975}$ becomes closer together. For very large n, $t_{.975} = 1.96$, the value from the standard normal distribution. For all practical purposes, t-distribution is equivalent to normal distribution for $n > 30$.

2.9 CHI-SQUARE (χ^2) DISTRIBUTION

Just as for each of the possible $\binom{N}{n}$ samples there is a mean $\overline{X}$, there is also a standard deviation S and a variance S^2. Whereas $\overline{X}$ has a normal (or approximately normal) distribution, so S^2 has a distribution. Specifically, we usually consider the value $(n - 1)\, S^2/\sigma^2$, which has what is called a *Chi-square* (χ^2) distribution.

Probabilities can be calculated for $(n - 1)\, S^2/\sigma^2$ (and hence for S^2), by finding the areas under the χ^2-distribution, the values of which are given in Appendix Table 3. Like t-distribution, χ^2-distribution is a family of distributions distinguished by values of degrees of freedom $f = n - 1$. If we wish to find the two values on the χ^2-distribution which have 95% of the distribution between them, then we call the smaller value $\chi^2_{.025}$ and the large value $\chi^2_{.975}$ and obtain the corresponding value from Appendix Table 3.

EXAMPLE: Using Appendix Table 3, the $\chi^2_{.025}$ and $\chi^2_{.975}$ values for degrees of freedom, $f = 9$ ($n = 10$, $f = n - 1 = 9$), are 2.20039 and 19.0228.

2.10 CONFIDENCE INTERVALS

A confidence interval aims at bracketing the true value of a population parameter, such as its mean or its standard deviation, by taking into account the uncertainty of the sample estimate of the parameter. Following we list the 95% confidence intervals (intervals which contain the unknown parameters with probability 0.95) for the most common and useful cases.

Confidence interval for the mean μ of a normal distribution, when the variance σ^2 is known.

$$\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

Confidence interval for a mean of a normal distribution, when the variance σ^2 is unknown and is estimated by S^2, the sample variance.

$$\left(\bar{X} - t_{.975}\frac{S}{\sqrt{n}}, \bar{X} + t_{.975}\frac{S}{\sqrt{n}}\right)$$

Where $t_{.975}$ is read from the t-distribution of Appendix Table 2.

Confidence interval for a proportion. If a discrete random variable has a binomial distribution, one might be concerned with estimating the population proportion of defectives, p. We take a random sample of size n and obtain X defectives, giving X/n as the proportion of defectives in the sample. If n is large and p is not too close to 1 or 0, the central limit theorem allows us to use normal approximation to binomial, and we have

$$\frac{X/n - p}{\sqrt{\frac{p(1-p)}{n}}}$$

approximately normally distributed. The 95% confidence interval for p is

$$\left(\frac{X}{n} - 1.96\sqrt{\frac{p(1-p)}{n}}, \frac{X}{n} + 1.96\sqrt{\frac{p(1-p)}{n}}\right)$$

This interval depends on p, which is, of course, unknown; however, if we replace p by X/n, we obtain the following approximate 95% confidence interval for p:

$$\left[\frac{X}{n} - 1.96\sqrt{\frac{\frac{X}{n}\left(1 - \frac{X}{n}\right)}{n}}, \frac{X}{n} + 1.96\sqrt{\frac{\frac{X}{n}\left(1 - \frac{X}{n}\right)}{n}}\right]$$

Confidence interval for variance σ^2 of a normal distribution. One calculates S^2, the sample variance based on a random sample of size n, and reads the values of $\chi^2_{.025}$ and $\chi^2_{.975}$ from Appendix Table 3 using n − 1 degrees of freedom. A 95% confidence interval is

$$\left(\frac{(n-1)S^2}{\chi^2_{.975}}, \frac{(n-1)S^2}{\chi^2_{.025}}\right)$$

A 95% confidence interval for σ, the standard deviation of a normal distribution is

$$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{.975}}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{.025}}}\right)$$

Confidence interval for the difference, $\mu_1 - \mu_2$, the means of two normal distributions. The first normal distribution has mean μ_1. A sample of n_1 observations from this distribution yields a sample mean $\bar{X}_1$ and a sample variance S_1^2. The second normal distribution has mean μ_2. A sample of n_2 observations from this distribution yields a sample means $\bar{X}_2$ and a sample variance S_2^2. The pooled variance is used for setting up confidence intervals and is given by

$$S_p^2 = \frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{n_1 + n_2 - 2}$$

The 95% confidence interval for $\mu_1 - \mu_2$ is

$$\left(\overline{X}_1 + \overline{X}_2 - t_{.975}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \overline{X}_1 - \overline{X}_2 + t_{.975}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

where the value of $t_{.975}$ is read from Appendix Table 2 for $n_1 + n_2 - 2$ degrees of freedom.

EXAMPLE: Calculate a 95% confidence interval for μ, σ^2, and σ using data and calculations of Sections 2.2 and 2.3.

$$\overline{X} = 3.32,\ S^2 = 0.1386,\ S = 0.3723,\ n = 60.$$

The 95% confidence interval for μ is

$$3.32 - (2.00)\,\frac{(0.3723)}{\sqrt{60}},\ 3.32 + (2.00)\,\frac{(0.3723)}{\sqrt{60}}$$

or (3.22, 3.42), where the value of $t_{.975}$ is approximately equal to 2.00 from Appendix Table 2. We are highly confident (95% confidence) that the mean skim milk moisture content lies between 3.22 and 3.42.

For the 95% confidence intervals for σ^2 and σ, we read χ^2 values with 59 degrees of freedom from Appendix Table 3 as follows:

$$\chi^2_{.975} = 83.30,\ \chi^2_{.025} = 40.48 \text{ (approximately).}$$

Thus, a 95% confidence interval for σ^2 is (0.098, 0.202) and a 95% confidence interval for σ is (0.313, 0.449).

2.11 TOLERANCE INTERVALS

A confidence interval brackets the population mean μ with a prechosen degree of confidence. However, in many practical problems, it is generally enough or even more informatory to set up an interval that will encompass a proportion p of the population with a prechosen degree of confidence. Such an interval is called a *tolerance interval*.

When the values of μ and σ are unknown and are estimated from sample values $\overline{X}$ and S respectively, the tolerance interval that encompasses at least a proportion p of the population with a 95% level of confidence is given by

$$(\overline{X} - K_2'\,(n, p)\, S,\ \overline{X} + K_2'\,(n, p)\, S)$$

where the values of coefficient K_2' (n, p) for various values of n and for p = 0.09, 0.95, and 0.99 are given in Appendix Table 4.

EXAMPLE: Using data and calculation of Sections 2.2 and 2.3, calculate a 95% tolerance interval that contains at least 95% of the population values.

$$\overline{X} = 3.32,\ S = 0.3723,\ n = 60,$$

K_2' (60, 0.95) = 2.33 from Appendix Table 4.

Thus, the tolerance interval is given by

$$[3.32 - (2.33)\,(0.3723),\ 3.32 + (2.33)\,(0.3723)]$$

or (2.34, 4.09). We are 95% confident that the interval will contain 95% of the population values.

2.12 DESIGNING AND PLANNING OF EXPERIMENTS

Designing an experiment means deciding how the observations or measurements should be taken to answer a particular question in a valid, efficient and economical way. The design and the final analysis go together; they are inseparable in the sense that if an experiment is properly designed, there will exist an appropriate way of analyzing the data. The following elements must be given careful consideration during the designing and planning of an experiment.

1. *Randomization:* Randomization is essential for a valid estimate of the experimental error and also to minimize bias in the results. The justification for randomization is that it makes the chance negligible that systematic differences between units receiving different treatments will persist in a long experiment and that it enables the error to be estimated whatever the form of uncontrolled variation. In effect the randomization rearranges the experimental units into random order and converts uncontrolled variation of whatever pattern into completely random variation. It is very important that randomization should cover all stages at which major errors may arise. Randomization is achieved by shuffling cards, etc. or, much more usually, by a table of random numbers. The method of using a random numbers table is explained in the next chapter.

2. *Replication:* The second essential feature of an experiment is replication. A treatment is repeated a number of times to obtain a more reliable estimate than is possible from a single observation. Since the error of the experiment arises from the differences between experimental units of the same treatment, that are not due to differences between the replicates, there is no other way but replication to get an estimate of the error of the experiment.

   Thus, the most effective way to increase the precision of an experiment is to increase the number of replications. Of course, replication beyond a limit may be impractical: since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, a decrease in $\sigma_{\bar{x}}$ is proportional to the square root of the number of replications — this is true if the variations due to replicates have been removed from error. The number of replications in a particular case depends on the variability of the material, cost of taking observations, etc. A rule-of-thumb is to get about 10 degrees of freedom for the experimental error; and generally one should not use less than four replications.

3. *Systematic error:* An important requirement for a good experiment is the absence of systematic error, i.e., to ensure that experimental units receiving one treatment differ in no systematic way from those receiving another treatment. Experimental units receiving one treatment should show only random differences from units receiving any other treatment, including the control, and should be allowed to respond independently of one another.

4. *Precision:* If the absence of systematic errors is achieved by randomization, the estimate of a treatment contrast obtained from the experiment will differ

its true value only by random errors. The probable magnitude of the random errors in the estimate of the treatment contrast can usually be measured by the standard error. The value of the standard error, and hence the precision of any particular experiment, will depend on (1) the intrinsic variability of the experimental material and the accuracy of the experimental work; (2) the number of experimental units (and on the number of repeat observations per experimental unit); and (3) the design of the experiment.

5. *Validity:* By validity of a sample design we mean that the sample should be so selected that the results could be interpreted objectively in terms of probability. In other words, valid tests or estimates about the population characteristics must be available. Also, the range of validity of conclusions should be very wide, without decreasing the accuracy of the experiment. The wider the range of conditions investigated in the experiment, the greater is the confidence we have in the extrapolation of the conclusions.
6. *Optimization:* The principle of optimization ensures that a given level of efficiency will be reached with minimum cost or that the maximum possible efficiency will be attained with a given level of cost. Efficiency is measured by the inverse of the sampling variance of the estimator and cost is measured by expenditure incurred in terms of money or man-hours.

In conclusion, the experiment should be simple in design, and a proper statistical analysis of the results should be possible without making artificial assumptions.

2.13 SAMPLE SIZE DETERMINATION

The determination of sample size is a problem that usually confronts the research worker in the initial stages of an investigation. The difficulty in solving most types of "sample-size" problems is that often we don't know what we want and we lack certain information necessary for calculations. To solve the problem of sample size, three questions must first be answered:

1. What variation is expected in the experiment?
2. What difference between the estimated and true value or what differences between treatments are expected?
3. What accuracy of estimation is desired?

Many short methods have been used for determining sample size such as extracting the square root of the number of boxes or taking a 10% sample. While these techniques are easy to use they are not based upon a statistical evaluation of the experiment.

When the frequency distribution of the variable X can be reasonably approximated by the normal distribution, the problem of sample size can be formulated using the confidence interval approach. Suppose that e is the error bound or tolerance on the sample mean, i.e., $|\overline{X} - \mu| = e$, and that we want to be 95% confident that the absolute difference $|\overline{X} - \mu|$ will be less than e. This means that

$$P[|\overline{X} - \mu| < e] = 0.95$$

i.e. $$P\left[|Z| < \frac{e}{\sigma_{\overline{X}}}\right] = 0.95$$

Hence

$$e = 1.96\, \sigma_{\bar{X}}$$

$$e = 1.96 \sqrt{\frac{N - n}{n\,(N - 1)}}\; \sigma$$

Simplifying, we get

$$n = \frac{N}{1 + \dfrac{e^2\,(N - 1)}{\sigma^2\,(1.96)^2}}$$

where n = sample size
N = lot size
z = normal deviate
= 1.96 for 95% confidence
e = error that the investigator is willing to tolerate between the estimated and the true value
and σ = standard deviation.

Note that if Z = 1.96 is taken approximately equal to 2, the formula can be written as:

$$n = \frac{N}{1 + \dfrac{e^2\,(N - 1)}{4\sigma^2}}$$

In cases where N is very large, the formula reduces to

$$n = \frac{4\sigma^2}{e^2}.$$

The corresponding sample size formulas for a proportion are:

$$n = \frac{N}{1 + \dfrac{e^2\,(N - 1)}{4pq}}, \text{ when N is small}$$

$$= \frac{4pq}{e^2}, \text{ when N is large}$$

where p = proportion defective,
and q = 1 − p = proportion not defective.

In cases where two treatments are compared, the sample size formula becomes:

$$n = \frac{8\sigma^2}{e^2} = \frac{8pq}{e^2}$$

As can be seen from these formulas, the precision of the estimate increases with the square root of the sample size. Also note that the population size N enters into the calculations only by multiplying by the adjustment factor $\sqrt{(N - n)/(N - 1)}$. This factor is close to one for large N. Thus, if a sample is selected randomly from the population, the size of the population is not important unless the sample is an appreciable proportion of it.

CHAPTER 3
SAMPLE SELECTION METHODS

3.1 SAMPLING THEORY

Sampling theory is a study of relationship between a population and samples drawn from the population. It is used to estimate unknown population parameters (such as population mean, variance, correlation, etc.) from a knowledge of corresponding sample quantities. Sampling theory is also useful in determining whether observed differences between two samples are actually due to chance variations or whether they are really significant. Such questions arise, for example, in testing a new serum for use in treatment of a disease or in deciding whether one production process is better than another.

In general, a study of inferences made concerning a population by use of samples drawn from it, together with indications of the accurancy of such inferences, is called statistical inference. In order that conclusions of sampling theory and statistical inference be valid, samples must be chosen to be *representative* of a population. One way in which a representative sample may be obtained is by a process called random sampling. A *random sample* may be defined as the sample drawn in such a way that the chance for inclusion of any item in the sample is predetermined. Such a chance is independent of the quality of the item and also independent of the items selected for the sample. When the chance for inclusion of any item in the sample is the same, it is referred to as *simple random sample*.

Though, it is important to select random representative samples, many practical situations do not lend themselves to such methods. The selection of a sample is generally influenced by the type and location of the lot. Broadly speaking, these are two commonly used methods of selecting a sample: *probability* or *random sampling*, and *nonprobability sampling*.

3.2 PROBABILITY SAMPLING

In probability sampling, the sample is drawn by random selection methods in accordance with the principles of statistical sampling and probability. The discretion of the sampler, with regard to which sampling units are to be included in the sample, is eliminated. Procedure for forming the estimates are automatic, being laid down beforehand as part of the sample design. With probability sampling, the precision of a particular probability sample can be estimated from the sample itself and the degree of precision stipulated. The four important sampling methods considered here are: simple random sampling, stratified random sampling, systematic sampling, and cluster sampling.

3.2.1 Simple random sampling

A simple random sample is selected from a lot or population using a random process and is the one in which all elements in the lot have an equal and independent chance of being included in the sample. Simple random samples can be drawn using tables of random numbers. Numerous random number tables are available for use. One is given in Appendix Table 5 and has been abstracted from the Rand Corporation's *A Million Random Digits with 100,000 Normal Deviates* (Free Press of Glenco, New York, 1955). To explain the use of random numbers, the first 30 sets of random numbers are taken from Appendix Table 5 and presented here in Table 3.1. For larger populations and repeated use of random numbers, consult the Rand Corporation tables.

DRAWING A SIMPLE RANDOM SAMPLE: Suppose a sample of eight boxes is to be drawn from a lot of 90 boxes. The boxes in the lot are labelled with numbers from 1 to 90.

TABLE 3.1 Random numbers

93108	77033	68325	10160	38667	62441
87023	94372	06164	30700	28271	08589
83279	48838	60935	70541	53814	95588
05832	80235	21841	35545	11148	34775
17308	88034	97765	35959	52843	44895

Starting at the top of the first column of Table 3.1., the boxes to be drawn for the sample are numbers

93 10 87 70 33 68 32 51

The number 93 is ignored since the corresponding box will not be formed in the lot. Proceeding along the row, we select one more random number, which is 01. (Note that the layout of numbers in groups of five within the table is simply for reading convenience). Thus, the final eight random numbers selected to make up the sample are

10 87 70 33 68 32 51 01

3.2.2 Stratified random sampling

A stratified random sample is one obtained by separating the population elements into some non-overlapping groups, called strata, and then selecting a simple random sample from each stratum. There are mainly three reasons stratified random sampling often results in increased information for a given cost:

1. The data is more homogeneous within each stratum than in the population as a whole.
2. The cost of conducting the actual sampling tends to be less for stratified random sampling than for simple random sampling because of administrative convenience.
3. When stratified sampling is used, separate estimates of population parameters can be obtained for each stratum without additional sampling.

Reduced variability within each stratum produces stratified sampling estimators which have smaller variances than do the correspondence simple random sampling estimators from the same sample size.

As an example, if in a shell egg packing station the boxes of eggs are placed on pallets according to the grade size, the population is divided into stratas and the sampling inspection can be carried out using stratified random sampling.

3.2.3 Systematic sampling

A sample obtained by randomly selecting from the first k elements (say) in the frame and every k th element thereafter is called a one in k-*systematic sample*. Consider N sampling units numbered serially from 1 to N from which a sample size of n is to be drawn. We find an integer k, called the sampling interval, such that $k = N/n$, and then select randomly a number c between 1 and k. Then the required systematic sample is given by

$$c, c + k, c + 2k, \ldots, c + (n - 1)k$$

Systematic sampling provides a useful alternative to simple random sampling in the sense that it is easier to perform, less subject to error, and provides greater information per unit cost.

As an example, a farmer can use a the in-ten systematic sample to determine the quality of maple syrup contained in the sap of trees on his farm, where the total number of trees, N, is unknown and he cannot conduct a simple random sample.

3.2.4 Cluster sampling

A cluster sample is a simple random sample in which each sampling unit is a collection, or cluster, of elements. The population is divided into clusters such that each cluster is designed to be as similar to each of the others as possible. The heterogeneity in the population is reflected within each cluster.

Cluster sampling is less costly than simple or stratified random sampling if the cost of obtaining a frame which lists all population elements is very high, or if the cost of obtaining observations increases as the distance separating the elements increases.

The first task in cluster sampling is to specify approximate clusters. Elements within a cluster are often physically close together and hence tend to have similar characteristics. Thus, the amount of information pertinent to a population parameter may not be increased substantially as new measurements are taken within a cluster. In general, the number of elements within a cluster will be small relative to the population size, and the number of clusters in the sample will be reasonably large.

To illustrate, suppose a Turkey Marketing Board wished to estimate the turkey buying of households in a small town. Travel costs from household to household are substantial. Therefore, the 5,000 households in the town are listed in 500 geographical clusters of 10 households each, and a simple random sample of 5 clusters is selected.

3.3 NON-PROBABILITY SAMPLING

In non-probability sampling, the investigator exercises discretion or judgement in deciding which items are to be included in the sample. Since no probabilities are assigned in regard to the selection of sampling units, some items from the population have a better chance of inclusion than others. Thus, the sampling biases cannot be calculated and the accuracy of the estimates cannot be determined. The most commonly used nonrandom sampling methods are: judgement sampling, quota sampling, restricted sampling, and convenience sampling.

3.3.1 Judgement sampling

In judgement sampling the investigators use their own judgement and experience regarding the population, lot, or sampling frame to decide which sample units to select. This method introduces nonuniformity in the process, and as such no statistical techniques can be applied to study the precision of the estimates.

Although judgement sampling is nonrandom, the practical difficulties of a particular situation, best comprehended by the sampler, may suggest this method as the only practical and feasible method of selecting the sample. Judgement is indispensable in any sampling procedure, and if investigators are experienced and skilled, and this method is carefully applied, judgemental sampling can yield valuable results.

3.3.2 Quota sampling

This method consists in specifying, in advance, quotas of the samples to be selected from different sampling frames or subgroups of lots. Sample selection is done by either judgement or random sampling. If the quotas are decided scientifically, and the sampling is done through probability sampling methods, the results can be reliable; otherwise this method can provide unreliable and inefficient results.

3.3.3 Restricted sampling

When a sample is taken from only a portion of the accessible lot or population, it is known as a restricted sample. The restriction can be natural, artificial, or purposely created for convenience. A restricted sample may not represent the population fully, and therefore may not provide meaningful generalizations for the entire population. However, in many situations, the physical placement of elements may create accessibility problems and inspection can only be carried out by taking convenient samples such as from a heavily loaded box car or heavily packed warehouse.

3.3.4 Convenience sampling

When an investigator selects a sample in any convenient way, the process is termed convenience sampling. Other terms such as *chunk sampling*, *grab sampling*, and *haphazard sampling* can also be used. In chunk or grab sampling

the investigator grabs a portion or chunk that is convenient; for example, the top layer in a box, units near the entrance, or first pallet. A haphazard sample is drawn in way the investigator likes. It is difficult to apply statistical techniques to such types of sampling methods.

3.4 BULK SAMPLING

Bulk sampling refers to the sampling of material which is available in bulk form. Bulk material may be gaseous, liquid or solid. The material may be homogeneous (non-segregated) like acid in a container or it may be segregated as is generally the case with bulk material occurring in nature, like solids and liquids shipped in large tanks, rail cars, or ships, or kept in stockpiles. The material may be in piles with no unique identifiable subdivisions that can be used as sampling units. It may also come in packages, bags or any material that may be subdivided into unique sampling units practicable for a routine sampling operation. Further, the material may be in *static* situation or *dynamic* situation.

Static situations include bulk heaps at a manufacturer's works; bulk load in transit in barges, rail and road wagons; bulk heaps or silos at farms or stores; etc. From pure sampling theory, it is impossible to obtain a representative sample from a static heap because one of the basic rules of sampling cannot be obeyed, i.e., every particle must have an equal chance of selection. Unless the whole heap can be passed through the sampling device, or can be coned and quartered completely, this cannot apply. Particles in the very center or on the bottom layer may have no chance at all of being selected. In addition, the problems of segregation in static heaps are well known and the segregation may affect the distribution of such characteristics as chemical composition, physical properties, etc. Segregations may occur because of size variation between particles or because of density variations between the constituents of a mix.

Dynamic situations include filling a bulk storage area at the factory; loading bulk transport; at the point of delivery, etc. In these situations, conveyor belts may be open to allow sampling either by mechanical or other means; or there may be a free fall position which will allow samples to be taken from the whole of the falling stream. Dynamic situations are simpler to handle and available sampling plans can be used.

3.4.1 Selecting samples of segregated material

Usually bulk material is sampled by taking increments of the material, blending these increments into a simple composite sample and then, if necessary, reducing this gross sample to a size suitable for laboratory testing. For a bulk material involving containers in batches, having known segregation pattern, a nested sampling plan is often appropriate. Such a plan calls for selecting a number of batches at random, selecting containers within these batches at random, and selecting random samples or increments from these containers. Where a material

is known to be stratified the plan may call for taking random samples or increments from each stratum or from a number of strata selected at random.

For example, in double stage sampling where the bulk consists of N primary units, with each unit composed of M possible increments, the sampling plan calls for selecting n primary units and from each of these a sample of m increments is taken. This gross sample is then reduced to a size suitable for laboratory testing.

Although very little statistical thinking has gone into the preparation of standard procedures for sampling of bulk material, some situations are well documented. One such case is the sampling of fertilizer. *The Official Methods of Analysis of the Association of Official Agricultural Chemists* gives the following directions for taking the original increments of fertilizer and forming the composite sampling:

> Use slotted single or double tube, or slotted tube and rod, with solid cone tip at one end. Take sample as follows: lay bag horizontally and remove core diagonally from end to end. From lots of 10 bags or more, take core from each of 10 bags. When necessary to sample lots of $<$ 10 bags, take cores but at least one core from each bag present. For bulk fertilizers, draw at least 10 cores from different regions. Bulk shipments may be sampled at time of loading or unloading by passing container through entire stream of material as it drops from transfer belt or chute. For small packages (10 pounds or less) take one entire package as sample. Reduce composite to quantity required, preferably by riffling, or by mixing thoroughly on clean oilcloth or paper and quartering. Place sample in airtight container.

To prepare the sample for laboratory analysis the directions run:

> Reduce gross sample to quantity sufficient for analysis or grind not $<$ 0.5 lb. of reduced sample without previous sieving. For fertilizer materials and moist fertilizer mixes, grind to pass sieve with 1 mm circular openings, or No. 29 std. sieve; for dry mixes that tend to segregate, grind to pass No. 40 std. sieve. Grind as rapidly as possible to avoid loss or gain of moisture during operation. Mix thoroughly and store in tightly stoppered bottles.

3.4.2 Objective of bulk sampling

The objectives may include one or more of the following:

1. To estimate the average value of a characteristic in a given lot of material and to establish confidence limits for this average;
2. To decide whether the average value for the lot meets a specification;
3. To obtain simultaneous estimates of the mean and variance or to make decisions based on combinations of these estimates.

Note that the formulation of theoretical models and estimation procedures for measuring variances involves understanding of techniques of advance mathematical statistics and have, thus, been excluded from the discussions in this book. However, some references are provided at the end for further reading on the subject of bulk sampling.

CHAPTER 4
ACCEPTANCE SAMPLING

4.1 INTRODUCTION

Acceptance sampling refers to the process of accepting or rejecting a lot by inspecting a sample selected in accordance with a predetermined sampling plan. A sampling plan specifies the amount of sampling to be done, the acceptance/rejection criteria, and the associated probabilities of acceptance. Sampling plans are based on several quality characteristics and the choice of a particular type of plan depends on the nature of the product and the purpose of inspection. Selecting an adequate and suitable sampling plan, though important, is not always an easy task, because the selection is dependent on a number of different factors such as ease of administration, protection afforded, amount of inspection required, cost of inspection, and the power of a plan to discriminate between a good and a bad lot. Before we discuss the methods of selecting the sampling plans, some important terms need to be defined.

4.2 METHODS OF INSPECTION

Measurement and evaluation of quality characteristics can be done by two methods, namely *attributes* and *variables*.

An attribute is a characteristic or property that is appraised in terms of whether it does or does not exist with respect to a given requirement. With attribute inspection, the units of product are classified as "defective" or "nondefective", "within tolerance" or "out-of-tolerance", or "correct" or "incorrect".

A variable is a characteristic or property that is appraised in terms of scalar values on a continuous scale. In variable inspection one measures quality characteristics such as weight, chemical purity, and temperature readings on a continuous numerical scale.

Attribute sampling plans are more commonly used methods of acceptance sampling than variable sampling plans because they are mathematically simple, economical, and administratively convenient. In this chapter we shall restrict our discussion to the derivation and usage of attribute sampling plans.

4.3 CHARACTERISTICS OF A SAMPLING PLAN

Every sampling plan is characterized by a *sample size* (n), an *acceptance number* (c), and the *probability of acceptance* (P_a). The sample size is the number of items selected for inspection. The acceptance number is the largest number of defectives (or defects) in the sample that will permit the product to be accepted. The probability of acceptance of a sampling plan is the percentage of samples out of a long series of samples that will cause the product to be

accepted. A complete plotting of the probability of acceptance for products at all possible levels of percent defective is known as an *operating characteristics (OC) curve*. One such OC curve is shown in Figure 4.1. It can be seen from the figure that the probability of acceptance for a product 2% defective is 0.95. This means that 95% of the product will be accepted.

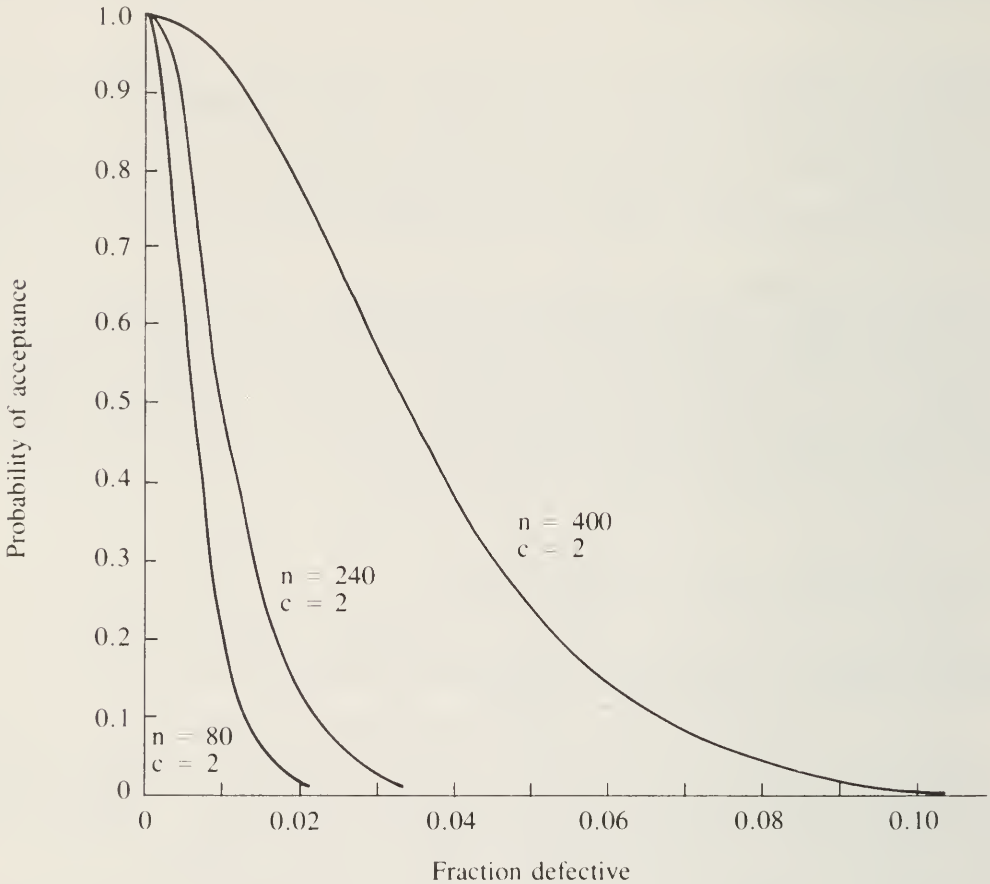


FIGURE 4.1 OC curves for samples of different size

4.4 PARAMETERS AFFECTING SAMPLING PLANS

Operating characteristic curves are a graphical means for showing the relationship between quality of lots submitted for sampling inspection and the probability of their acceptance. The steepness of the slope and shape of an OC curve are dependent on the sample size (n) and the acceptance number (c). The steepness of the OC curve indicates the power of the sampling plan to discriminate between good and bad quality. An increase in sample size results in a steepening of the OC curve as indicated in Figure 4.1.

The effect of changing the acceptance number is illustrated in Figure 4.2. The larger the acceptance number, the poorer the quality that will be passed by the sampling plan. The effect of increasing the acceptance number is to shift the location of the entire OC curve to the right. To obtain a sampling plan

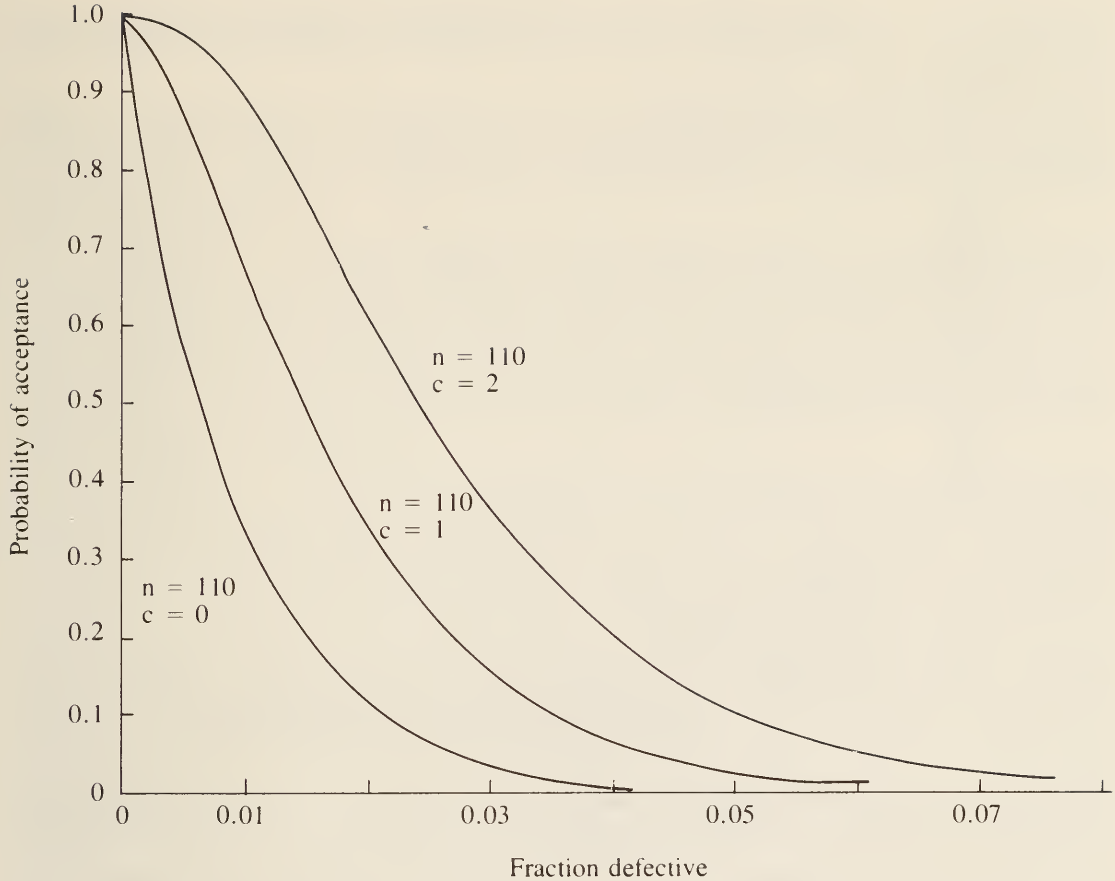


FIGURE 4.2 OC curves for different acceptance numbers

that closely satisfies the quality requirements, the sample size and acceptance number have to be simultaneously adjusted.

The effect of the lot size (N) on a sampling plan is not too pronounced. The relationship between the lot size (N) and sample size (n) for a specific method of selecting the sample was discussed in previous chapters.

4.5 CONSTRUCTION OF AN OC CURVE

Calculations of acceptance probabilities are done by using the hypergeometric, binomial, or Poisson distributions. We shall make use of the Poisson approximation to binomial for the derivation of these probabilities. Appendix Table 6 gives the summation of terms of Poisson's exponential binomial limit.

EXAMPLE 4.1: Suppose we wish to calculate the probabilities of acceptance and draw the OC curve when

$$n = 240, \quad \text{and} \quad c = 2$$

Set up the table headings as in Table 4.1.

TABLE 4.1 Preparation for calculating acceptance probabilities in example 4.1

n	np′	p′	P_a
240			0.99
240			0.95
240			0.75
240			0.50
240			0.25
240			0.10
240			0.02
240			0.01

where n = sample size = 240
p' = fraction defective
np' = number defective
and P_a = probability of acceptance = suitably chosen coordinates to facilite construction of the OC curve

Now (using Appendix Table 6), find the np' values for $c = 2$ corresponding to the P_a values closest to the P_a values chosen in Table 4.1. For example, searching through the table, we find $np' = 0.44$ for $P_a = 0.99$ under the column for $c = 2$. Similarly, $np' = 1.7$ for $P_a = 0.757$ which is the closest value to $P_a = 0.75$ set in Table 4.1. The same procedure is followed for each of the other P_a values and we obtain Table 4.2.

TABLE 4.2 Acceptance probabilities for example 4.1

n	np′	p′	P_a
240	0.44	0.0018	0.990
240	0.82	0.0042	0.950
240	1.70	0.0071	0.757
240	0.70	0.0113	0.494
240	3.90	0.0163	0.253
240	5.30	0.0221	0.102
240	7.50	0.0313	0.020
240	8.40	0.0350	0.010

The OC curve for Table 4.2 is plotted in Figure 4.1.

4.6 TYPE OF LOT SAMPLING PLANS

There are three important types of lot sampling plans: *single, double* and *multiple*. The choice of which type to use depends on many factors such as quality history, quality requirements, economy, and the particular sampling inspection situation.

Single sampling: In a single sampling plan a single sample of n items is selected at random from the lot. The decision concerning the acceptability of the lot is made on the basis of results obtained from the sample. If the number of defectives found in the sample is less than or equal to the acceptance number c, the lot is accepted. If the number of defectives is equal to or greater than the rejection number r, the lot is rejected.

Double sampling: In a double sampling plan a first sample of n_1 units is selected at random from the lot and inspected. If the numbers of defectives is less than or equal to the first acceptance number c_1, the lot is accepted. If the number of defectives is equal to or greater than the first rejection number r_1, the lot is rejected. If no decision can be made from the first sample because the number of defectives is greater than c_1, but less than r_1, a second sample of n_2 units is selected at random from the lot and inspected. If the cumulative number of defectives from the first and second sample is less than or equal to the second acceptance number c_2, the lot is accepted. And if the cumulative number of defectives is equal to or greater than the second number r_2, the lot is rejected.

The average number of items inspected with double sampling is generally less that inspected with single sampling. Some inspectors consider double sampling equivalent to giving another chance to the product. Despite a smaller sampling rate, double sampling is less frequently used than single sampling.

Multiple sampling: The procedure in multiple sampling is similar to that in double sampling except that the number of successive samples required to reach a decision to accept or reject the lot may be more than two. The number of steps required to reach a firm decision depends on the cumulative number of defectives found in the samples taken progressively. There are acceptance/rejection criteria for each step — accept at any step where the cumulative defectives are equal to or less than the acceptance number, and reject where the defectives equal or exceed the rejection number for that step. If the number of defectives is between the accept/reject figures, we take another sample. All multiple sampling is terminated after a specified number of steps by arranging the acceptance and rejection figures to be consecutive at the last step, thus forcing a decision to accept or reject the lot. The size of the cumulative sample at the last step is larger than the equivalent in single and double plans.

4.7 RISKS AND QUALITY INDICES

Regardless of the inspection plan used (sampling or 100% inspection), there is always a risk that a small percentage of defective units will be passed. Two types of risks are involved in sampling: risk of rejecting a good lot (*producer's risk* or α (alpha) risk), and risk of accepting a bad lot (*consumer's risk* or β (beta) risk).

The OC curve of a sampling plan quantifies these risks and makes it possible to state them numerically and describe the quantities of product that can be expected to be accepted if the quality standard is met, and the quantity rejected if the standard is not met. Ideally, we would like to have a sampling plan as illustrated in Figure 4.3 when it is desired to accept all lots 3.0% defective or less and reject all lots having a quality level greater than 3.0% defective. All lots less than 3.0% defective will be accepted 100% of the time and all lots greater than 3.0% defective will be rejected 100% of the time. In practice, no sampling plan can discriminate perfectly; there is always the risk of rejecting good lots, and accepting bad lots. We always start with an OC curve (Figure 4.3) for an actual sampling plan and try to make the slope of the curve steeper by improving the quality of incoming lots.

Associated with the producer's risk is the quantity called *acceptable quality level* (AQL), defined as the maximum percent defective (or maximum number of defects per 100 units) that, for the purpose of sampling inspection, can be considered as a process average. Since the acceptance/rejection of a lot is based on the inspection of a sample and involves the inherent risks associated with it, the producer is generally concerned about the probability of rejecting lots that meet the specifications and would not want a high-quality lot to be rejected more than 5% of the time. Thus, the producer and the consumer agree on an AQL value and the associated α risk. Consequently, a lot which is of AQL quality is accepted with a probability of $(1 - \alpha)$ or P_A. This gives us a point (AQL, P_A) on the OC curve.

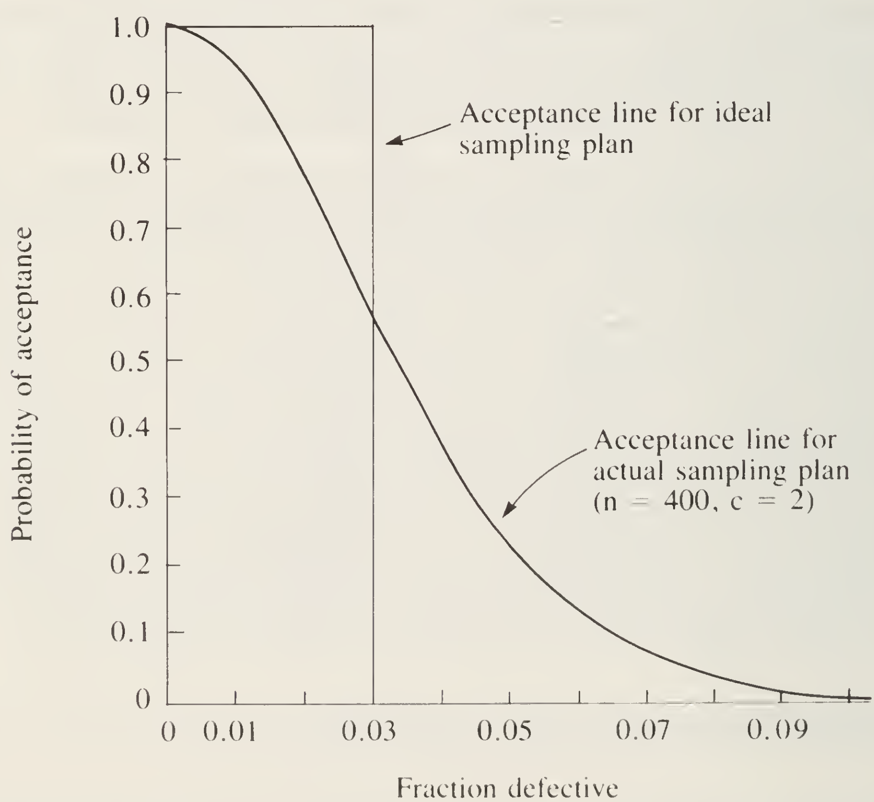


FIGURE 4.3 OC curves for ideal and actual sampling plans

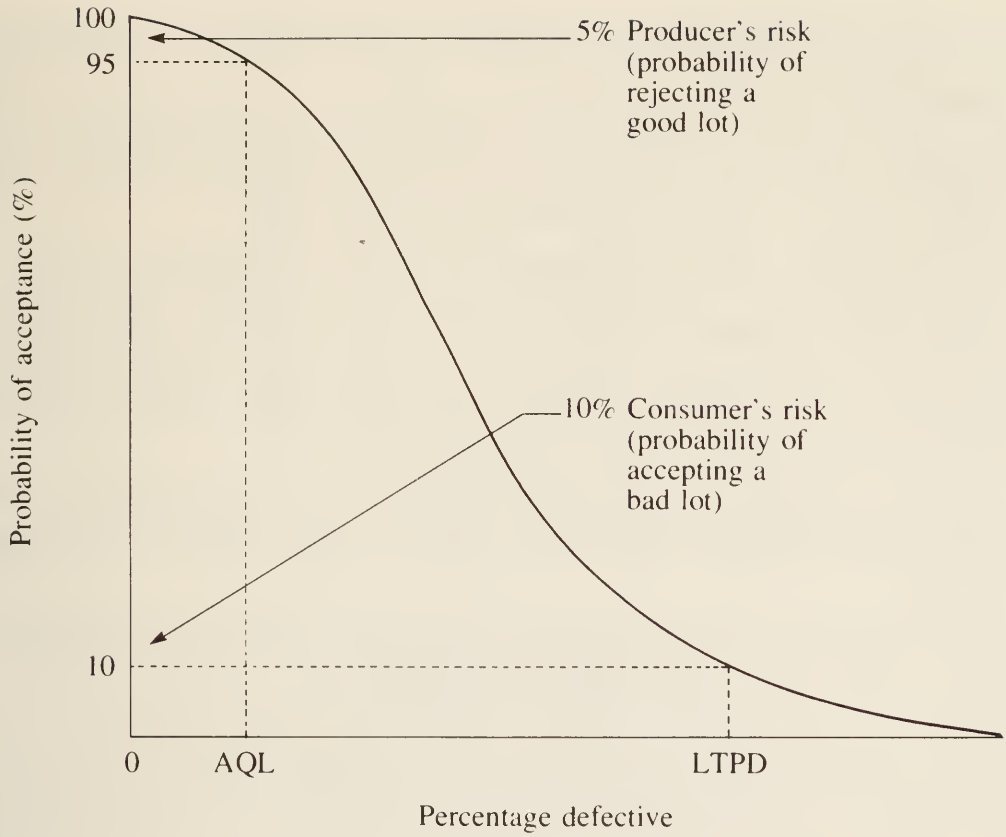


FIGURE 4.4 OC curve for attributes sampling

The consumer, on the other hand, also sets a certain quality level P_t for the incoming lot or process, called *lot tolerance percent defective* (LTPD). This is the worst quality consumers would be willing to accept. It would be rarely accepted, or accepted with a probability of β or P_c, say 10%. Thus we have another point (P_t, P_c) on the OC curve.

These two points (AQL, P_a) and (P_t, P_c) determine the shape of the OC curve for a sampling plan and are illustrated in Figure 4.4. The OC curve that comes closest to passing through these two points, giving n and c, is the most suitable sampling plan for that situation. Because of the integral nature of these criteria, an exact solution to the desired specifications set by producer and consumer may not be realized. It is often necessary to satisfy one party and almost satisfy the other in selecting a sampling plan.

4.8 CHOOSING A SAMPLING PLAN

A sampling plan can be derived using Appendix Table 6 and the two points (AQL,P_a) and (P_t, P_c). Once the values of these four characteristics are established, the OC curve is manipulated around these points until an optimum sampling plan results. This procedure is explained in the next section by considering an example.

Fortunately, we do not have to go through the rigor of deriving a sampling plan with the two points (AQL, P_A) and (P_t, P_c) every time. There are ready-made sampling tables available that can be used. The two most commonly used are the *U.S. Department of Defense Military Standard 105D (Mil Std 105D)*, or its equivalent, *Canadian Government Specifications Board CGSB-105-GP-1*, based on AQLs, and *Dodge* and *Romig* tables based on LTPDs.

The AQL sampling plans are designed to protect the producer, whereas the LTPD sampling plans are designed to protect the consumer. Other types of sampling plans designed to protect the consumer and given in Dodge and Romig tables are known as *average outgoing quality limit* (AOQL) sampling plans. For a full description of LTPD and AOQL sampling plans, consult the Dodge and Romig tables cited in the bibliography.

In this text, we restrict our discussion to the use of CGSB-105-GP-1 (AQL sampling plans) as they are the most widely accepted methods of acceptance sampling.

EXAMPLE 4.2: *Derivation of a single sampling plan:* Suppose a food processing company wishes to establish a single sampling plan for sampling inspection of cans of mixed vegetables. The producer agrees with the buyer (consumer) on an AQL value of 2% defective with $\alpha = 0.05$ or acceptance probability of 0.95. The buyer also specifies a LTPD value of 8% with $\beta = 0.10$ or acceptance probability of 0.10. This means that the producer wants to be 95% sure that the lot will be accepted if it meets the AQL specification of 2%, and the buyer wants to ascertain that the incoming lot will be accepted only 10% of the time if it is 8% or more defective. Thus, we must establish a value of n and c that will meet these specifications.

Using Appendix Table 6 the values of $np'_{0.95}$ and $np'_{0.10}$ are obtained for various values of c and the ratios $np'_{0.10}/np'_{0.95}$ are computed. The calculations are shown in Table 4.3. Next calculate the ratio

$$\frac{nPt_{0.10}}{nAQL_{0.95}} = \frac{0.08}{0.02} = 4.0$$

This value, when compared with the corresponding ratios in Table 4.3, lies between c of 4 and 5. Now consider four sets of probabilities resulting from combining each of c with LTPD and AQL values to calculate the corresponding n values. Calculations for the four sets of values are:

(a) When $c = 4$, $P_A = 0.95$

$$n = \frac{np'_{0.95}}{AQL} = \frac{1.97}{0.02} = 98$$

$$np' = n \times P_t = 98 \times 0.08 = 7.84$$

Thus, for $np' = 7.84$, and $c = 4$, $P_c = 0.112$ from Appendix Table 6.

(b) When $c = 4$, $P_c = 0.10$

$$n = \frac{np'_{0.10}}{P_t} = \frac{8.0}{0.08} = 100$$

$$np' = n \times AQL = 100 \times 0.02 = 2.0$$

TABLE 4.3 Acceptance calculations for example 4.2

c	$np'_{0.95}$	$np'_{0.10}$	$np'_{0.10}/np'_{0.95}$
0	0.05	2.3	45.10
1	0.35	3.8	10.96
2	0.82	5.3	6.50
3	1.37	6.7	4.89
4	1.97	8.0	4.06
5	2.60	9.3	3.55
6	3.30	10.5	3.21
7	4.00	11.8	2.96
8	4.70	13.0	2.77

Thus, for $np' = 2.00$, and $c = 4$, $P_A = 0.947$ from Appendix Table 6.

(c) When $c = 5$, $P_A = 0.95$

$$n = \frac{np'_{0.95}}{AQL} = \frac{2.60}{0.02} = 130$$

$$np' = n \times P_t = 130 \times 0.08 = 10.4$$

Thus, for $np' = 10.4$, and $c = 5$, $P_c = 0.053$ from Appendix Table 6.

(c) When $c = 5$, $P_c = 0.10$

$$n = \frac{np'_{0.10}}{P_t} = \frac{9.5}{0.08} = 116$$

$$np' = n \times AQL = 116 \times 0.02 = 2.32$$

Thus, for $np' = 2.32$, and $c = 5$, $P_A = 0.97$ from Appendix Table 6.

The next step is to analyze the four sampling plans and choose the one that fits the given specifications most closely. In doing so, we can see that plan (a) gives the required protection for the producer, but increases the probability of accepting lots of quality equal to P_t. Plan (b) gives the required protection for the consumer and gives almost exact probability of accepting good lots at the 95% level. Plan (c) gives required protection to the producer but reduces the risk of accepting poor lots. Plan (d) gives the required protection to the consumer but increases the probability of accepting good lots.

Since plan (b) gives assurance at almost exact specifications, this is the plan of choice. If a sample of 100 items is selected and inspected, the lot is accepted if four or less items are found defective. If the number of defectives is greater than four, the lot is rejected. The OC curve for the plan of choice (n, c = 100, 4) is shown in Figure 4.5.

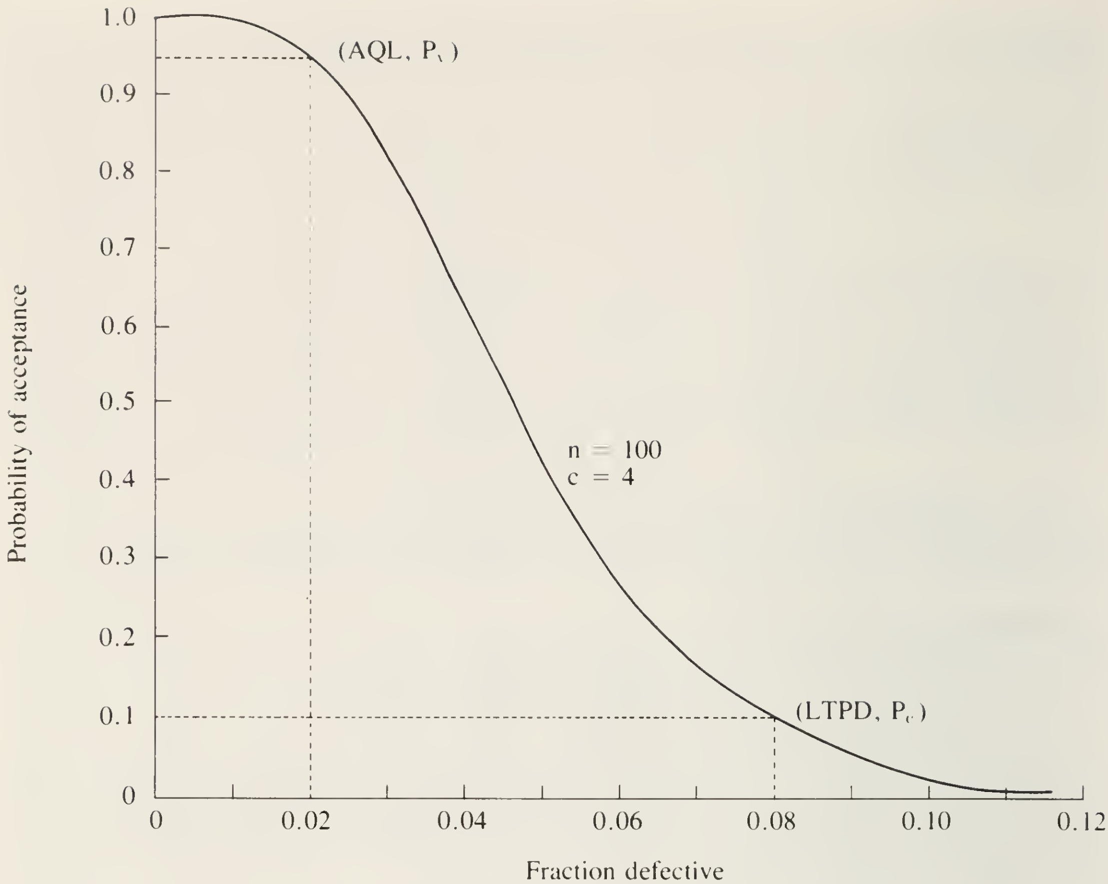


FIGURE 4.5 OC curve for single sampling plan

4.9 CANADIAN GOVERNMENT SPECIFICATIONS BOARD CGSB-105-GP-1 (or MIL. STD. 105-D)

The *United States Department of Defense Mil Std 105D*, entitled *Sampling Procedures and Tables for Inspection by Attributes*, is published in two other forms: *Canadian Government Specification Board Standard CGBS-105-GP-1*, and *ABC Standard Sampling Procedures and Tables for Inspection by Attributes*. It represents the international standard agreed upon by the United States, Britian, and Canada. All three standards are identical with the exception of their titles.

CGSB-105-GP-1 provides sampling inspection plans by attributes based on AQLs designed to be applied to lots emerging from long production runs of many units of products. Information is also provided to allow for easy extraction of the protection afforded by these plans in terms of LTPD and AOQL.

Reproduction of sample pages from the CGSB-105-GP-1 are given in Section 4.10.

The basic aim of the standard is the maintenance of the outgoing quality level at a given "acceptable quality level" or better. It is designed so that if the

producer runs consistently at precisely the AQL, then the great majority of his lots can be expected to pass. Thus, the AQL is the minimum quality performance at which the producer may safely run; he is, therefore, advised to run at or better than the AQL.

Three types of sampling plans are provided in this standard: single, double, and multiple. The size of sample required for inspection is dependent on four factors: inspection level, lot size, type of sampling, and AQL.

Seven inspection levels, S-1, S-2, S-3, S-4, I, II, and III, are provided for varying degrees of discrimination and each level provides different sample sizes for a given lot size. In the order given above, sample size (therefore discrimination) increases from a minimum at level S-1 to a maximum at level III. Levels S-1 to S-4 are considered special levels, which are limited to situations where it is imperative that only small sizes be used, such as destructive testing of expensive units of product. Inspection level II is the normal level and is to be used at the commencement of inspection activities unless otherwise specified.

The standard also provides three levels of inspection in terms of the severity of inspection: normal, tightened, and reduced. Normal inspection is used to start. Then, if the quality is shown to be poor, the inspector is directed to be more severe with his inspection and uses the tightened level. If the quality is shown to be consistently high, reduced inspection is indicated. Guidelines for switching rules between normal, tightened, and reduced inspections are provided.

A letter code system is used for determining sample size. This is given in Table 1 of CGSB-105-GP-1. The letter assigned to a given sample is dependent on the inspection level and lot size. In the table, varying blocks of lot sizes are listed vertically and inspection levels listed horizontally. Any specific lot size can be associated with a corresponding block listed in the table.

The next step is deciding the type of sampling plan and the AQL to be used. The standard provides single, double, and multiple sampling plans for normal, tightened, and reduced inspection levels. The AQLs progress in steps from a minimum of 0.010 to a maximum of 1000. AQLs from 0.010 up to 10 may be expressed in either percent defective or defects per 100 units. AQLs greater than 10 are expressed in defects per 100 units only.

Let us now summarize the steps for finding a specific sampling acceptance plan in the Standard.

1. Decide on the size of lot N, which is to be sample inspected. This need not be a production lot size.
2. Decide upon an inspection level, in general II.
3. Using 1 and 2, enter Table I to find the corresponding sample size code letter, A, B, . . . , R, the last calling for largest sample sizes.
4. Décide upon single, double, or multiple sampling.
5. Decide whether to start with normal (almost always), tightened, or reduced sampling.

6. Table II-A, III-A, or IV-A will thus be determined by 4 and 5 (if normal inspection is to be used).
7. Decide upon the inspection basis on defectives or defects.
8. Decide upon the desired AQL: for percent defectives 10.0 or less or defects per 100 units; using only what is available in tables.
9. Enter the table determined in 6, using the AQL for the column from 8 and the row from the sample size code letter in 3. This commonly gives the acceptance-rejection numbers in the block, the sample size(s) being given to the left of this block.
10. Following the above, we may reach an asterisk or an arrow. An asterisk means to use the single sampling plan for the desired AQL and code letter, instead of double or multiple sampling. If an arrow is encountered, follow it to the first block with acceptance-rejection numbers, using sample sizes to the left of this block, not to the left of the original block.

For example, suppose we have lot sizes of 250 units and will use inspection level II, for normal sampling with an AQL of 0.40% defectives. From Table I we find the sample size code letter G for a lot size of 250. For normal single sampling, we use Table II. In this table in the 0.40 column and the G row, we find AC = 0, Re = 1, and the sample size for code letter G as 32. Thus, the plan calls for taking a random sample of 32 from the 250 in the lot, inspect, and finding d defective (1) accept if $d = 0$, (2) reject if $d \geqslant 1$.

Under the same conditions but for double inspection, we use Table III-A, and find $n_1 = n_2 = 20$, and a dot (.) in the block under the 0.40 column. The dot (.) indicates two alternatives (1) either to use the corresponding single sampling plan or (2) to use double sampling plan below, where available in Table III-A. Following it down to the row, we find $AC_1 = 0$, $Re_1 = 2$, $AC_2 = 1$, $Re_2 = 2$. Now going over to the left of this block we find $n_1 = n_2 = 80$. Note very particularly that we do not use the sample sizes $n_1 = n_2 = 20$, which are normally used for code letter G. Thus our plan is to take a random sample of 80 from the 250 in the lot, inspect, and find d_1 defectives (1) accept if $d_1 = 0$, (2) reject if $d_1 \geqslant 2$, or (3) take another sample if $d_1 = 1$. Now this second sample of 80 from the 170 remaining in the lot yield d_2 defectives. Then (1) accept the lot if $d_1 + d_2 = 1$ or (2) reject if $d_1 + d_2 \geqslant 2$.

NOTE: In reduced sample tables, you would come across a gap between acceptance and rejection number, for example, AC = 0 and Re = 2. Now, if our inspection reveals 1 defective, this seems to mean that no decision is reached. This is covered is Section 10.1.4 of the Standard. There it says that if such an event occurs, we will accept this lot (because the previous quality has been excellent relative to the AQL), but we are now alerted to the possibility that quality has slipped from its previous excellence. Therefore, we abandon reduced sampling with its quite lenient OC curve and go back to normal sampling.

Now let us have a look at Table X-G, which we have included as a sample. There is in the Standard such a pair of pages for each code letter A to R. They give a wealth of information on the respective sampling plans for each code

letter. All sample plans — single, double, and multiple — are listed on the second pages as on the page, X-G-2. At the top of the first page X-G-1 are given the OC curves for all AQL's for single normal inspection. Most are calculated using the Poisson distribution as in our Appendix Table 6, but the binomial is used if the single plan has $n \leq 80$ and also the $AQL \leq 10.0$. As usual, P_a is the vertical scale, the bottom scale for percent defective or defects per 100 units for the process. The OC curve for the normal single sampling plan for code letter G and AQL = 0.40%, namely, $n = 32$, $Ac = 0$, $Re = 1$ is shown in the graph next to the number "0.40". Thus, for example, if the product is 2% defective, $P_a = .54$.

The table in the lower half of the first page gives certain convenient points on the OC curve, namely, in percents $P_{.99}$, $P_{.95}$, $P_{.90}$, and so on to $P_{.01}$. These figures are for single normal sampling plan, as are the OC curves. The OC curves for double and multiple normal sampling "are matched as closely as practicable."

Let us now describe the remaining tables which we have reproduced. Only the normal and tightened multiple sampling plans IV-A, IV-B, have been included, omitting the corresponding reduced plans.

Table V-B provides the average outgoing quality limits, AOQL's, for all tightened, single-sampling plans. This table thus gives a measure of the consumer protection provided when on tightened inspection. Except for those single tightened plans for which the acceptance number $Ac = 0$, all of these AOQL's are at about the corresponding AQL or better. This means that when we go onto tightened inspection we are, in general, using a plan providing outgoing quality averaging no worse than the AQL.

Table VI-A and VI-B give quality levels having a 0.10 probability of acceptance if offered, respectively, in terms of percent defective and defects per 100 units. These are particularly useful in case we are sample inspecting isolated or infrequent lots, since these figures tell what quality levels, of which a given plan provides a 90% assurance against acceptance, are offered.

Table IX contains very useful information on the average sample number (ASN) curves, in a convenient compact form, with which we may compare single, double, and multiple sampling. They are for both normal and tightened inspection, but do not include cases for which the single plan has $Ac = 0$. All other acceptance numbers for a single plan are included.

This completes our discussion of the standard with its aim of maintaining outgoing quality at the AQL or better, but still giving a small risk of a lot being rejected when the producer is running at the AQL or better.

4.10 SAMPLE PAGES FROM CGSB-105-GP-1

105-GP-1
6 January 1964

CANADIAN GOVERNMENT SPECIFICATIONS BOARD

STANDARD

ON

INSPECTION BY ATTRIBUTES

1. SCOPE

1.1 Purpose - This standard establishes sampling plans and procedures for inspection by attributes. When specified by the responsible authority, this standard shall be referenced in the specification, contract, inspection instructions, or other documents, and the provisions set forth herein shall govern. The "responsible authority" shall be designated in one of the above documents.

1.2 Application - Sampling plans designated in this standard are applicable, but not limited, to inspection of the following:

(a) End items.

(b) Components and raw materials.

(c) Operations.

(d) Materials in process.

(e) Supplies in storage.

(f) Maintenance operations.

(g) Data or records.

(h) Administrative procedures.

These plans are intended primarily to be used for a continuing series of lots or batches. The plans may also be used for the inspection of isolated lots or batches, but in this latter case the user is cautioned to consult the operating characteristic curves to find a plan that will yield the desired protection (see 11.6).

1.3 Inspection - Inspection is the process of measuring, examining, testing or otherwise comparing the unit of product (see 1.5) with the requirements.

1.4 Inspection by Attributes - Inspection by attributes is inspection whereby either the unit of product is classified simply as defective or nondefective, or the number of defects in the unit of product is counted, with respect to a given requirement or set of requirements.

1.5 Unit of Product - The unit of product is the thing inspected in order to determine its classification as defective or nondefective, or to count the number of defects. It may be a single article, a pair, a set, a length, an area, an operation, a volume, a component of an end product, or the end product itself. The unit of product may or may not be the same as the unit of purchase, supply, production or shipment.

2. CLASSIFICATION OF DEFECTS AND DEFECTIVES

2.1 Method of Classifying Defects - A classification of defects is the enumeration of possible defects of the unit of product classified according to their seriousness. A defect is any nonconformance of the unit of product with specified requirements. Defects will normally be grouped into one or more of the following classes, defects may, however, be grouped into other classes, or into subclasses within these classes.

2.1.1 Critical Defect - A critical defect is a defect that judgment and experience indicate is likely to result in hazardous or unsafe conditions for individuals using, maintaining or depending upon the product; or a defect that judgment and experience indicate is likely to prevent performance of the tactical or planned function of a major end item such as a ship, aircraft, tank, missile, space vehicle, refrigerator, radio set, etc. NOTE: For a special provision relating to critical defects, see 6.3.

2.1.2 Major Defect - A major defect is a defect, other than critical, that is likely to result in failure, or to reduce materially the usability of the unit of product for its intended purpose.

2.1.3 Minor Defect - A minor defect is a defect that is not likely to reduce materially the usability of the unit of product for its intended purpose, or is a departure from established standards having little bearing on the effective use or operation of the unit.

2.2 Method of Classifying Defectives - A defective is a unit of product that contains one or more defects. Defectives will usually be classified as follows:

2.2.1 Critical Defective - A critical defective contains one or more critical defects and may also contain major and or minor defects. NOTE: For a special provision relating to critical defectives, see 6.3.

2.2.2 Major Defective - A major defective contains one or more major defects, and may also contain minor defects but no critical defect.

2.2.3 Minor Defective - A minor defective contains one or more minor defects but no critical nor major defect.

3. PER CENT DEFECTIVE AND DEFECTS PER HUNDRED UNITS

3.1 Expression of Nonconformance - The extent of nonconformance of product shall be expressed either in terms of per cent defective or in terms of defects per hundred units.

3.2 Per cent Defective - The per cent defective of any given quantity of units of product is one hundred times the number of defective units of product contained therein divided by the total number of units of product:

$$\text{Per cent defective} = \frac{\text{Number of defective units}}{\text{Number of units inspected}} \times 100$$

3.3 Defects per Hundred Units - The number of defects per hundred units of any given quantity of units of product is one hundred times the number of defects contained therein (one or more defects being possible in any unit of product) divided by the total number of units of product:

$$\text{Defects per hundred units} = \frac{\text{Number of defects}}{\text{Number of units inspected}} \times 100$$

4. ACCEPTABLE QUALITY LEVEL (AQL)

4.1 Use - The AQL, together with the Sample Size Code Letter, is used for indexing the sampling plans provided herein.

4.2 Definition - The AQL is the maximum per cent defective (or the maximum number of defects per hundred units) that, for purposes of sampling inspection, can be considered satisfactory as a process average (see 11.2).

4.3 Note on the Meaning of AQL - When a consumer designates some specific value of AQL for a certain defect or group of defects, he indicates to the supplier that his (the consumer's) acceptance sampling plan will accept the great majority of the lots of batches that the supplier submits, provided the process average level of per cent defective (or defects per hundred units) in these lots or batches is no greater than the designated value of AQL. Thus, the AQL is a designated value of per cent defective (or defects per hundred units) in the lots, that the consumer indicates will be accepted most of the time by the acceptance sampling procedure to be used. The sampling plans provided herein are so arranged that the probability of acceptance at the designated AQL value depends upon the sample size, being generally higher for large samples than for small ones, for a given AQL. The AQL alone does not describe the protection to the consumer for individual lots or batches but more directly relates to what might be expected from a series of lots or batches, provided the steps indicated in this publication are taken. It is necessary to refer to the operating characteristic curve of the plan to determine what protection the consumer will have.

4.4 Limitation - The designation of an AQL shall not imply that the supplier has the right to supply knowingly any defective unit of product.

4.5 Specifying AQL's - The AQL to be used will be designated in the contract or by the responsible authority. Different AQL's may be designated for groups of defects considered collectively, or for individual defects. An AQL for a group of defects may be designated in addition to AQL's for individual defects, or subgroups, within that group. AQL values of 10.0 or less may be expressed either in per cent defective or in defects per hundred units; those over 10.0 shall be expressed in defects per hundred units only.

4.6 Preferred AQL's - The values of AQL's given in these tables are known as preferred AQL's. If, for any product, an AQL is designated other than a preferred AQL, these tables are not applicable.

5. SUBMISSION OF PRODUCT

5.1 Lot or Batch - The term "lot" or "batch" shall mean "inspection lot" or "inspection batch," i.e., a collection of units of product from which a sample is to be drawn and inspected to determine conformance with the acceptability criteria, and may differ from a collection of units designated as a lot or batch for other purposes (e.g., production, shipment, etc.).

5.2 Formation of Lots or Batches - The product shall be assembled into identifiable lots, sublots, batches, subbatches, or in such other manner as may be prescribed (see 5.4). Each lot or batch shall, as far as is practicable, consist of units of product of a single type, grade, class, size and composition, manufactured under essentially the same conditions and at essentially the same time.

5.3 Lot or Batch Size - The lot or batch size is the number of units of product in a lot or batch.

5.4 Presentation of Lots or Batches - The formation of the lots or batches, lot or batch size, and the manner in which each lot or batch is to be presented and identified by the supplier shall be designated or approved by the responsible authority. As necessary, the supplier shall provide adequate and suitable storage space for each lot or batch, equipment needed for proper identification and presentation, and personnel for all handling of product required for drawing of samples.

6. ACCEPTANCE AND REJECTION

6.1 Acceptability of Lots or Batches - Acceptability of a lot or batch will be determined by the use of a sampling plan or plans associated with the designated AQL or AQL's.

6.2 Defective Units - The right is reserved to reject any unit of product found defective during inspection whether that unit of product forms part of a sample or not, and whether the lot or batch as a whole is accepted or rejected. Rejected units may be repaired or corrected and resubmitted for inspection with the approval of, and in the manner specified by, the responsible authority.

6.3 Special Reservation for Critical Defects - The supplier may be required, at the discretion of the responsible authority, to inspect every unit of the lot or batch for critical defects. The right is reserved to inspect every unit submitted by the supplier for critical defects, and to reject the lot or batch immediately, when a critical defect is found. The right is reserved also to sample, for critical defects, every lot or batch submitted by the supplier and to reject any lot or batch if a sample drawn therefrom is found to contain one or more critical defects.

6.4 Resubmitted Lots or Batches - Lots or batches found unacceptable shall be resubmitted for reinspection only after all units are re-examined or retested, and all defective units are removed or defects corrected. The responsible authority shall determine whether

normal or tightened inspection shall be used, and whether reinspection shall include all types or classes of defects or only the particular types or classes of defects that caused initial rejection.

7. DRAWING OF SAMPLES

7.1 Sample - A sample consists of one or more units of product drawn from a lot or batch, the units of the sample being selected at random without regard to their quality. The number of units of product in the sample is the sample size.

7.2 Representative Sampling - When appropriate, the number of units in the sample shall be selected in proportion to the size of sublots or subbatches, or parts of the lot or batch, identified by some rational criterion. When representative sampling is used, the units from each part of the lot or batch shall be selected at random.

7.3 Time of Sampling - Samples may be drawn after all the units comprising the lot or batch have been assembled, or during assembly of the lot or batch.

7.4 Double or Multiple Sampling - When double or multiple sampling is to be used, each sample shall be selected over the entire lot or batch excluding previous samples.

8. NORMAL, TIGHTENED AND REDUCED INSPECTION

8.1 Initiation of Inspection - Normal inspection shall be used at the start of inspection unless otherwise directed by the responsible authority.

8.2 Continuation of Inspection - Normal, tightened or reduced inspection shall continue unchanged for each class of defects or defectives on successive lots or batches except where the switching procedures given below require a change. The switching procedures shall be applied to each class of defects or defectives independently.

8.3 Switching Procedures

8.3.1 Normal to Tightened - When normal inspection is in effect, tightened inspection shall be instituted when 2 out of 5 consecutive lots or batches have been rejected on original inspection (i.e., ignoring resubmitted lots or batches for this procedure).

8.3.2 Tightened to Normal - When tightened inspection is in effect, normal inspection shall be instituted when 5 consecutive lots or batches have been considered acceptable on original inspection.

8.3.3 Normal to Reduced - When normal inspection is in effect, reduced inspection shall be instituted providing that all of the following conditions are satisfied:

(a) The preceding 10 lots or batches (or more, as indicated by the note to Table VIII) have been on normal inspection and none has been rejected on original inspection.

(b) The total number of defectives (or defects), in the samples from the preceding 10 lots or batches (or such other number as was used for condition (a) above) is equal to or less than the applicable number given in Table VIII. If double or multiple sampling is in use, all samples inspected should be included, not "first" samples only.

(c) Production is at a steady rate.

(d) Reduced inspection is considered desirable by the responsible authority.

8.3.4 Reduced to Normal - When reduced inspection is in effect, normal inspection shall be instituted if any of the following occurs on original inspection:

(a) A lot or batch is rejected.

(b) A lot or batch is considered acceptable under the procedures of 10.1.4.

(c) Production becomes irregular or delayed.

(d) Other conditions warrant that normal inspection shall be instituted.

8.4 Discontinuation of Inspection - In the event that 10 consecutive lots or batches remain on tightened inspection (or such other number as may be designated by the responsible authority), inspection under the provisions of this document should be discontinued pending action to improve the quality of submitted material.

9. SAMPLING PLANS

9.1 Sampling Plan - A sampling plan indicates the number of units of product from each lot or batch that are to be inspected (sample size or series of sample sizes) and the criteria for determining the acceptability of the lot or batch (acceptance and rejection numbers).

9.2 Inspection Level - The inspection level determines the relationship between the lot or batch size and the sample size. The inspection level to be used for any particular requirement will be prescribed by the responsible authority. Three inspection levels, I, II and III, are given in Table I for general use. Unless otherwise specified, Inspection Level II will be used. Inspection Level I may, however, be specified when less discrimination is needed, or Level III may be specified for greater discrimination. Four additional special levels, S-1, S-2, S-3 and S-4, are given in the same table and may be used where relatively small sample sizes are necessary and large sampling risks can or must be tolerated.

NOTE: In the designation of inspection levels S-1 to S-4, care must be exercised to avoid AQL's inconsistent with these inspection levels.

9.3 Code Letters - Sample sizes are designated by code letters. Table I shall be used to find the applicable code letter for the particular lot or batch size and the prescribed inspection level.

9.4 Obtaining Sampling Plan - The AQL and the code letter shall be used to obtain the sampling plan from Tables II, III or IV. When no sampling plan is available for a given combination of AQL and code letter, the tables direct the user to a different letter. The sample size to be used is given by the new code letter, not by the original letter. If this procedure leads to different sample sizes for different classes of defects, the code letter corresponding to the largest sample size derived may be used for all classes of defects when designated or approved by the responsible authority. As an alternative to a single sampling plan with an acceptance number of 0, the plan with an acceptance number of 1, with its correspondingly larger sample size for a designated AQL (where available), may be used when designated or approved by the responsible authority.

9.5 Types of Sampling Plans - Three types of sampling plans, Single, Double and Multiple, are given in Tables II, III and IV, respectively. When several types of plans are available for a given AQL and code letter, any one may be used. A decision as to type of plan, either single, double, or multiple, when available for a given AQL and code letter, will usually be based upon the comparison between the administrative difficulty and the average sample sizes of the available plans. The average sample size of multiple plans is less than for double (except in the case corresponding to single acceptance number 1) and both of these are always less than a single sample size. Usually the administrative difficulty for single sampling and the cost per unit of the sample are less than for double or multiple.

10. DETERMINATION OF ACCEPTABILITY

10.1 Per cent Defective Inspection - To determine acceptability of a lot or batch under per cent defective inspection, the applicable sampling plan shall be used in accordance with 10.1.1, 10.1.2, 10.1.3 and 10.1.4.

10.1.1 Single Sampling Plan - The number of sample units inspected shall be equal to the sample size given by the plan. If the number of defectives found in the sample is equal to or less than the acceptance number, the lot or batch shall be considered acceptable. If the number of defectives is equal to or greater than the rejection number, the lot or batch shall be rejected.

10.1.2 Double Sampling Plan - The number of sample units first inspected shall be equal to the first sample size given by the plan. If the number of defectives found in the first sample is equal to or less than the first acceptance number, the lot or batch shall be considered acceptable. If the number of defectives found in the first sample is equal to or greater than the first rejection number, the lot or batch shall be rejected. If the number of defectives found in the first sample is between the first acceptance and rejection numbers, a second sample of the size given by the plan shall be inspected. The number of defectives found in the first and second samples shall be accumulated. If the cumulative number of defectives is equal to or less than the second acceptance number, the lot or batch shall be considered acceptable. If the cumulative number of defectives is equal to or greater than the second rejection number, the lot or batch shall be rejected.

10.1.3 Multiple Sample Plan - Under multiple sampling, the procedure shall be similar to that specified in 10.1.2, except that the number of successive samples required to reach a decision may be more than two.

10.1.4 Special Procedure for Reduced Inspection - Under reduced inspection, the sampling procedure may terminate without either acceptance or rejection criteria having been met. In these circumstances, the lot or batch will be considered acceptable, but normal inspection will be reinstated starting with the next lot or batch (see 8.3.4 (b)).

10.2 Defects Per Hundred Units Inspection - To determine the acceptability of a lot or batch under Defects per Hundred Units inspection, the procedure specified for Per cent Defective inspection (10.1) shall be used, except that the word "defects" shall be substituted for "defectives."

11. SUPPLEMENTARY INFORMATION

11.1 Operating Characteristic Curves - The operating characteristic curves for normal inspection shown in Table X (pages 30 - 62) indicate the percentage of lots or batches that may be expected to be accepted under the various sampling plans for a given process quality. The curves shown are for single sampling; curves for double and multiple sampling are matched as closely as practicable. The O.C. curves shown for AQL's greater than 10.0 are based on the Poisson distribution and are applicable for defects per hundred units Inspection; those for AQL's of 10.0 or less and sample sizes of 80 or less are based on the binomial distribution and are applicable for per cent defective inspection; those for AQL's of 10.0 or less and sample sizes larger then 80 are based on the Poisson distribution and are applicable either for defects per hundred units inspection, or for per cent defective inspection (the Poisson distribution being an adequate approximation to the binomial distribution under these conditions). Tabulated values, corresponding to selected values of probabilities of acceptance (P_a, in per cent) are given for each of the curves shown and, in addition, for tightened inspection, and for defects per hundred units for AQL's of 10.0 or less and sample sizes of 80 or less.

11.2 Process Average - The process average is the average per cent defective or average number of defects per hundred units (whichever is applicable) of product submitted by the supplier for original inspection. Original inspection is the first inspection of a particular quantity of product as distinguished from the inspection of product that has been resubmitted after prior rejection.

11.3 Average Outgoing Quality (AOQ) - The AOQ is the average quality of outgoing product including all accepted lots or batches, plus all rejected lots or batches after the rejected lots or batches have been effectively 100 per cent inspected and all defectives replaced by nondefectives.

11.4 Average Outgoing Quality Limit (AOQL) - The AOQL is the maximum of the AOQ's for all possible incoming qualities for a given acceptance sampling plan. AOQL values are given in Table V-A for each of the single sampling plans for normal inspection, and in Table V B for each of the single sampling plans for tightened inspection.

11.5 Average Sample Size Curves - Average sample size curves for double and multiple sampling are in Table IX. These show the average sample sizes that may be expected to occur under the various sampling plans for a given process quality. The curves assume no curtailment of inspection and are approximate to the extent that they are based upon the Poisson distribution, and that the sample sizes for double and multiple sampling are assumed to be 0.631n and 0.25n respectively, where n is the equivalent single sample size.

11.6 Limiting Quality Protection - The sampling plans and associated procedures given in this publication were designed for use where the units of product are produced in a continuing series of lots or batches over a period of time. However, if the lot or batch is of an isolated nature, it is desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that provide not less than a specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a Limiting Quality (LQ) and a consumer's risk to be associated with it. Tables VI and VII give values of LQ for the commonly used consumer's risks of 10 per cent and 5 per cent respectively. If a different value of consumer's risk is required, the O.C. curves and their tabulated values may be used. The concept of LQ may also be useful in specifying the AQL and Inspection Levels for a series of lots or batches, thus fixing minimum sample size where there is some reason for avoiding (with more than a given consumer's risk) more than a limiting proportion of defectives (or defects) in any single lot or batch.

TABLE I—Sample size code letters

(See 9.2 and 9.3)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	A	B
9	to	15	A	A	A	A	A	B	C
16	to	25	A	A	B	B	B	C	D
26	to	50	A	B	B	C	C	D	E
51	to	90	B	B	C	C	C	E	F
91	to	150	B	B	C	D	D	F	G
151	to	280	B	C	D	E	E	G	H
281	to	500	B	C	D	E	F	H	J
501	to	1200	C	C	E	F	G	J	K
1201	to	3200	C	D	E	G	H	K	L
3201	to	10000	C	D	F	G	J	L	M
10001	to	35000	C	D	F	H	K	M	N
35001	to	150000	D	E	G	J	L	N	P
150001	to	500000	D	E	G	J	M	P	Q
500001	and	over	D	E	H	K	N	Q	R

TABLE II-A—Single sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample size	Acceptable Quality Levels (normal inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

⇩ = Use first sampling plan below arrow. If sample size equals, or exceeds lot or batch size, do 100 percent inspection.

⇧ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

SINGLE NORMAL

TABLE II-B — Single sampling plans for tightened inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample size	Acceptable Quality Levels (tightened inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow.
Ac = Acceptance number
Re = Rejection number.

TABLE II-C—Single sampling plans for reduced inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample size	Acceptable Quality Levels (reduced inspection)†																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
C	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑
D	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑
E	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑	↑
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑
G	13	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑
H	20	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑
J	32	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	50	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	80	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	125	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	200	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	315	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	500	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	800	↑	↑	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow.
Ac = Acceptance number.
Re = Rejection number.
† = If the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection (see 10.1.4).

TABLE III-A—Double sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	Acceptable Quality Levels (normal inspection)																									
				0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	•	•	•	•	•	•	•	•	•
B	First	2	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31
	Second	2	4																	1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57
C	First	3	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑
	Second	3	6																1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57	
D	First	5	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑	↑
	Second	5	10															1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57		
E	First	8	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑	↑	↑
	Second	8	16														1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57			
F	First	13	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑
	Second	13	26													1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27						
G	First	20	20	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑
	Second	20	40												1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27							
H	First	32	32	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑
	Second	32	64											1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27								
J	First	50	50	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	50	100										1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27									
K	First	80	80	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	80	160									1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27										
L	First	125	125	↓	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	125	250								1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27											
M	First	200	200	↓	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	200	400							1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27												
N	First	315	315	↓	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	315	630						1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27													
P	First	500	500	↓	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	500	1000					1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27														
Q	First	800	800	•	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	800	1600				1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27															
R	First	1250	1250	↑	↑	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	1250	2500			1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27																

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow

Ac = Acceptance number

Re = Rejection number

• = Use corresponding single sampling plan (or alternatively, use double sampling plan below, where available)

DOUBLE NORMAL

TABLE III-B — Double sampling plans for tightened inspection (Master table) (See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	•	•	•	•	•	•	•
B	First	2	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	15 20	23 29
	Second	2	4																		1 2	3 4	4 5	6 7	11 12	15 16	23 24	34 35	52 53
C	First	3	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	15 20	23 29	↑
	Second	3	6																	1 2	3 4	4 5	6 7	11 12	15 16	23 24	34 35	52 53	
D	First	5	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	15 20	23 29	↑	↑
	Second	5	10																1 2	3 4	4 5	6 7	11 12	15 16	23 24	34 35	52 53		
E	First	8	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	15 20	23 29	↑	↑	↑
	Second	8	16															1 2	3 4	4 5	6 7	11 12	15 16	23 24	34 35	52 53			
F	First	13	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑
	Second	13	26														1 2	3 4	4 5	6 7	11 12	15 16	23 24						
G	First	20	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑
	Second	20	40													1 2	3 4	4 5	6 7	11 12	15 16	23 24							
H	First	32	32	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑
	Second	32	64												1 2	3 4	4 5	6 7	11 12	15 16	23 24								
J	First	50	50	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	50	100											1 2	3 4	4 5	6 7	11 12	15 16	23 24									
K	First	80	80	↓	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	80	160										1 2	3 4	4 5	6 7	11 12	15 16	23 24										
L	First	125	125	↓	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	125	250									1 2	3 4	4 5	6 7	11 12	15 16	23 24											
M	First	200	200	↓	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	200	400								1 2	3 4	4 5	6 7	11 12	15 16	23 24												
N	First	315	315	↓	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	315	630							1 2	3 4	4 5	6 7	11 12	15 16	23 24													
P	First	500	500	↓	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	500	1000						1 2	3 4	4 5	6 7	11 12	15 16	23 24														
Q	First	800	800	↓	•	↓	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	800	1600					1 2	3 4	4 5	6 7	11 12	15 16	23 24															
R	First	1250	1250	•	↑	↓	0 2	0 3	1 4	2 5	3 7	6 10	9 14	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	1250	2500				1 2	3 4	4 5	6 7	11 12	15 16	23 24																
S	First	2000	2000			0 2																							
	Second	2000	4000			1 2																							

Acceptable Quality Levels (tightened inspection)

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number

Re = Rejection number

• = Use corresponding single sampling plan (or, alternatively, use double sampling plan below, where available).

TABLE III-C — Double sampling plans for reduced inspection (Master table)

(See 9.4 and 9.5)

Acceptable Quality Levels (reduced inspection)†

Sample size code letter	Sample	Sample size	Cumulative sample size	0.010 Ac Re	0.015 Ac Re	0.025 Ac Re	0.040 Ac Re	0.065 Ac Re	0.10 Ac Re	0.15 Ac Re	0.25 Ac Re	0.40 Ac Re	0.65 Ac Re	1.0 Ac Re	1.5 Ac Re	2.5 Ac Re	4.0 Ac Re	6.5 Ac Re	10 Ac Re	15 Ac Re	25 Ac Re	40 Ac Re	65 Ac Re	100 Ac Re	150 Ac Re	250 Ac Re	400 Ac Re	650 Ac Re	1000 Ac Re
A				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	•	•	•	•	•	•	•	•	•
B																	•	↑		•	•	•	•	•	•	•	•	•	•
C																•	↑	↓	•	•	•	•	•	•	•	•	•	•	↑
D	First	2	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	7 12	11 17	↑	↑
	Second	2	4															0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16	18 22	26 30		
E	First	3	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	7 12	11 17	↑	↑	↑
	Second	3	6														0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16	18 22	26 30			
F	First	5	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑
	Second	5	10													0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16						
G	First	8	8	↓	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑
	Second	8	16												0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16							
H	First	13	13	↓	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑
	Second	13	26											0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16								
J	First	20	20	↓	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	20	40										0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16									
K	First	32	32	↓	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	32	64									0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16										
L	First	50	50	↓	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	50	100								0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16											
M	First	80	80	↓	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	80	160							0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16												
N	First	125	125	↓	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	125	250						0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16													
P	First	200	200	↓	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	200	400					0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16														
Q	First	315	315	•	↑	↓	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	315	630				0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16															
R	First	500	500	↑	↑	0 2	0 3	0 4	0 4	1 5	2 7	3 8	5 10	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	500	1000			0 2	0 4	1 5	3 6	4 7	6 9	8 12	12 16																

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow.
Ac = Acceptance number.
Re = Rejection number.
• = Use corresponding single sampling plan (or alternatively, use double sampling plan below, when available.)
† = If, after the second sample, the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection (see 10.14).

DOUBLE
REDUCED

TABLE IV-A—Multiple sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	Acceptable Quality Levels (normal inspection)																									
				0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A				⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇩	⇩	•	•	•	•	•	•	•	•	•
B																	•	⇧		↔	↔	↔	↔	↔	↔	↔	↔	↔	↔
C																•	⇧	⇩	↔	↔	↔	↔	↔	↔	↔	↔	↔	↔	⇧
D	First	2	2	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	4 12	6 16	⇧	⇧
	Second	2	4															* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14	11 19	17 27		
	Third	2	6															0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19	19 27	29 39		
	Fourth	2	8															0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25	27 34	40 49		
	Fifth	2	10															1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29	36 40	53 58		
	Sixth	2	12															1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33	45 47	65 68		
	Seventh	2	14															2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38	53 54	77 78		
E	First	3	3	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	4 12	6 16	⇧	⇧	⇧
	Second	3	6														* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14	11 19	17 27			
	Third	3	9														0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19	19 27	29 39			
	Fourth	3	12														0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25	27 34	40 49			
	Fifth	3	15														1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29	36 40	53 58			
	Sixth	3	18														1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33	45 47	65 68			
	Seventh	3	21														2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38	53 54	77 78			
F	First	5	5	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	⇧	⇧	⇧	⇧	⇧	⇧
	Second	5	10													* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14						
	Third	5	15													0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19						
	Fourth	5	20													0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25						
	Fifth	5	25													1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29						
	Sixth	5	30													1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33						
	Seventh	5	35													2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38						
G	First	8	8	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	⇧	⇧	⇧	⇧	⇧	⇧	⇧
	Second	8	16												* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14							
	Third	8	24												0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19							
	Fourth	8	32												0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25							
	Fifth	8	40												1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29							
	Sixth	8	48												1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33							
	Seventh	8	56												2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38							
H	First	13	13	⇩	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	⇧	⇧	⇧	⇧	⇧	⇧	⇧	⇧
	Second	13	26											* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14								
	Third	13	39											0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19								
	Fourth	13	52											0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25								
	Fifth	13	65											1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29								
	Sixth	13	78											1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33								
	Seventh	13	91											2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38								
J	First	20	20	⇩	⇩	⇩	⇩	⇩	⇩	•	⇧	⇩	* 2	* 2	* 3	* 4	0 4	0 5	1 7	2 9	⇧	⇧	⇧	⇧	⇧	⇧	⇧	⇧	⇧
	Second	20	40										* 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14									
	Third	20	60										0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19									
	Fourth	20	80										0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25									
	Fifth	20	100										1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29									
	Sixth	20	120										1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33									
	Seventh	20	140										2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38									

⇩ = Use first sampling plan below arrow (refer to continuation of table on following page, when necessary). If sample size equals or exceeds lot or batch size, do 100 percent inspection.
⇧ = Use first sampling plan above arrow
Ac = Acceptance number
Re = Rejection number
• = Use corresponding single sampling plan (or alternatively, use multiple sampling plan below, where available)
↔ = Use corresponding double sampling plan (or alternatively, use multiple sampling plan below, where available).
* = Acceptance not permitted at this sample size

TABLE IV-A — Multiple sampling plans for normal inspection (Master table)
(Continued)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
K	First	32	32	↓	↓	↓	↓	↓	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	32	64									# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14										
	Third	32	96									0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19										
	Fourth	32	128									0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25										
	Fifth	32	160									1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29										
	Sixth	32	192									1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33										
	Seventh	32	224									2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38										
L	First	50	50	↓	↓	↓	↓	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	50	100								# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14											
	Third	50	150								0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19											
	Fourth	50	200								0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25											
	Fifth	50	250								1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29											
	Sixth	50	300								1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33											
	Seventh	50	350								2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38											
M	First	80	80	↓	↓	↓	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	80	160							# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14												
	Third	80	240							0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19												
	Fourth	80	320							0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25												
	Fifth	80	400							1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29												
	Sixth	80	480							1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33												
	Seventh	80	560							2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38												
N	First	125	125	↓	↓	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	125	250						# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14													
	Third	125	375						0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19													
	Fourth	125	500						0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25													
	Fifth	125	625						1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29													
	Sixth	125	750						1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33													
	Seventh	125	875						2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38													
P	First	200	200	↓	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	200	400					# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14														
	Third	200	600					0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19														
	Fourth	200	800					0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25														
	Fifth	200	1000					1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29														
	Sixth	200	1200					1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33														
	Seventh	200	1400					2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38														
Q	First	315	315	•	↑	↓	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	315	630				# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14															
	Third	315	945				0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19															
	Fourth	315	1260				0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25															
	Fifth	315	1575				1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29															
	Sixth	315	1890				1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33															
	Seventh	315	2205				2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38															
R	First	500	500	↑	↑	# 2	# 2	# 3	# 4	0 4	0 5	1 7	2 9	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	500	1000			# 2	0 3	0 3	1 5	1 6	3 8	4 10	7 14																
	Third	500	1500			0 2	0 3	1 4	2 6	3 8	6 10	8 13	13 19																
	Fourth	500	2000			0 3	1 4	2 5	3 7	5 10	8 13	12 17	19 25																
	Fifth	500	2500			1 3	2 4	3 6	5 8	7 11	11 15	17 20	25 29																
	Sixth	500	3000			1 3	3 5	4 6	7 9	10 12	14 17	21 23	31 33																
	Seventh	500	3500			2 3	4 5	6 7	9 10	13 14	18 19	25 26	37 38																

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow (refer to preceding page, when necessary).
Ac = Acceptance number
Re = Rejection number
• = Use corresponding single sampling plan (or alternatively, use multiple plan below, where available).
= Acceptance not permitted at this sample size

MULTIPLE NORMAL

TABLE IV-B—Multiple sampling plans for tightened inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	Acceptable Quality Levels (tightened inspection)																									
				0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	•	•	•	•	•	•	•
B				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	↔	↔	↔	↔	↔	↔	↔	↔	↔
C				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	↔	↔	↔	↔	↔	↔	↔	↔	↔	↑
D	First	2	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	3 10	6 15	↑	↑
	Second	2	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	10 17	16 25	↑	↑
	Third	2	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	17 24	26 36	↑	↑
	Fourth	2	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	24 31	37 46	↑	↑
	Fifth	2	10	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	32 37	49 55	↑	↑
	Sixth	2	12	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	40 43	61 64	↑	↑
	Seventh	2	14	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	48 49	72 73	↑	↑
E	First	3	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	3 10	6 15	↑	↑	↑
	Second	3	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	10 17	16 25	↑	↑	↑
	Third	3	9	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	17 24	26 36	↑	↑	↑
	Fourth	3	12	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	24 31	37 46	↑	↑	↑
	Fifth	3	15	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	32 37	49 55	↑	↑	↑
	Sixth	3	18	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	40 43	61 64	↑	↑	↑
	Seventh	3	21	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	48 49	72 73	↑	↑	↑
F	First	5	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑
	Second	5	10	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑
	Third	5	15	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑
	Fourth	5	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑
	Fifth	5	25	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑
	Sixth	5	30	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑
	Seventh	5	35	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑
G	First	8	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑
	Second	8	16	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑
	Third	8	24	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑
	Fourth	8	32	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑
	Fifth	8	40	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑
	Sixth	8	48	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑
	Seventh	8	56	↓	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑
H	First	13	13	↓	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑
	Second	13	26	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑
	Third	13	39	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	13	52	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	13	65	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	13	78	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	13	91	↓	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑
J	First	20	20	↓	↓	↓	↓	↓	↓	↓	•	↓	↓	# 2	# 2	# 3	# 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	20	40	↓	↓	↓	↓	↓	↓	↓		↓	↓	# 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	20	60	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	20	80	↓	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	20	100	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	20	120	↓	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	20	140	↓	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow (refer to continuation of table on following page when necessary). If sample size equals or exceeds lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow
Ac = Acceptance number
Re = Rejection number
• = Use corresponding single sampling plan (or alternatively, use multiple sampling plan below where available)
↔ = Use corresponding double sampling plan (or alternatively, use multiple sampling plan below, where available)
= Acceptance not permitted at this sample size

MULTIPLE TIGHTENED

18

TABLE IV-B — Multiple sampling plans for tightened inspection (Master table) (Continued)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Acceptable Quality Levels (tightened inspection)																									
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
K	First	32	32	↓	↓	↓	↓	↓	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	32	64	↓	↓	↓	↓	↓	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	32	96	↓	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	32	128	↓	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	32	160	↓	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	32	192	↓	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	32	224	↓	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	First	50	50	↓	↓	↓	↓	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	50	100	↓	↓	↓	↓	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	50	150	↓	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	50	200	↓	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	50	250	↓	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	50	300	↓	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	50	350	↓	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	First	80	80	↓	↓	↓	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	80	160	↓	↓	↓	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	80	240	↓	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	80	320	↓	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	80	400	↓	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	80	480	↓	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	80	560	↓	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	First	125	125	↓	↓	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	125	250	↓	↓	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	125	375	↓	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	125	500	↓	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	125	625	↓	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	125	750	↓	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	125	875	↓	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	First	200	200	↓	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	200	400	↓	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	200	600	↓	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	200	800	↓	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	200	1000	↓	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	200	1200	↓	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	200	1400	↓	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	First	315	315	↓	•	↓	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	315	630	↓		↓	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	315	945	↓		↓	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	315	1260	↓		↓	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	315	1575	↓		↓	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	315	1890	↓		↓	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	315	2205	↓		↓	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	First	500	500	•	↑	↓	* 2	* 2	* 3	* 4	0 4	0 6	1 8	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Second	500	1000		↑	↓	* 2	0 3	0 3	1 5	2 7	3 9	6 12	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Third	500	1500		↑	↓	0 2	0 3	1 4	2 6	4 9	7 12	11 17	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fourth	500	2000		↑	↓	0 3	1 4	2 5	3 7	6 11	10 15	16 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Fifth	500	2500		↑	↓	1 3	2 4	3 6	5 8	9 12	14 17	22 25	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Sixth	500	3000		↑	↓	1 3	3 5	4 6	7 9	12 14	18 20	27 29	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
	Seventh	500	3500		↑	↓	2 3	4 5	6 7	9 10	14 15	21 22	32 33	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	First	800	800			* 2																							
	Second	800	1600			* 2																							
	Third	800	2400			0 2																							
	Fourth	800	3200			0 3																							
	Fifth	800	4000			1 3																							
	Sixth	800	4800			1 3																							
	Seventh	800	5600			2 3																							

⇩ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

⇧ = Use first sampling plan above arrow (refer to preceding page, when necessary).

Ac = Acceptance number

Re = Rejection number

• = Use corresponding single sampling plan (or alternatively, use multiple sampling plan below, where available).

* = **Acceptance not permitted at this sample size**

MULTIPLE
TIGHTENED

TABLE V-B — *Average Outgoing Quality Limit Factors for Tightened Inspection (Single sampling)*

(See 11.4)

Code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Acceptable Quality Level																									
A	2																			42	69	97	160	260	400	620	970
B	3															12			28	46	65	110	170	270	410	650	1100
C	5														7.4			17	27	39	63	100	160	250	390	610	
D	8													4.6			11	17	24	40	64	99	160	240	380		
E	13												2.8			6.5	11	15	24	40	61	95	150	240			
F	20											1.8			4.2	6.9	9.7	16	26	40	62						
G	32										1.2			2.6	4.3	6.1	9.9	16	25	39							
H	50									0.74			1.7	2.7	3.9	6.3	10	16	25								
J	80								0.46			1.1	1.7	2.4	4.0	6.4	9.9	16									
K	125							0.29			0.67	1.1	1.6	2.5	4.1	6.4	9.9										
L	200						0.18			0.42	0.69	0.97	1.6	2.6	4.0	6.2											
M	315					0.12			0.27	0.44	0.62	1.0	1.6	2.5	3.9												
N	500				0.074			0.17	0.27	0.39	0.63	1.0	1.6	2.5													
P	800			0.046			0.11	0.17	0.24	0.40	0.64	0.99	1.6														
Q	1250		0.029			0.067	0.11	0.16	0.25	0.41	0.64	0.99															
R	2000	0.018			0.042	0.069	0.097	0.16	0.26	0.40	0.62																
S	3150			0.027																							

Note: For the exact AOQL, the above values must be multiplied by $\left(1 - \frac{\text{Sample size}}{\text{Lot or Batch size}}\right)$ (see 11.4)

AOQL TIGHTENED

LQ (DEFECTIVES)
10.0%

TABLE VI-A — *Limiting Quality (in percent defective) for which P_a = 10 Percent*
(for Normal Inspection, Single sampling)

(See 11.6)

Code letter	Sample size	Acceptable Quality Level															
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10
A	2															68	
B	3														54		
C	5													37			58
D	8												25			41	54
E	13											16			27	36	44
F	20										11			18	25	30	42
G	32									6.9			12	16	20	27	34
H	50								4.5			7.6	10	13	18	22	29
J	80							2.8			4.8	6.5	8.2	11	14	19	24
K	125						1.8			3.1	4.3	5.4	7.4	9.4	12	16	23
L	200					1.2			2.0	2.7	3.3	4.6	5.9	7.7	10	14	
M	315				0.73			1.2	1.7	2.1	2.9	3.7	4.9	6.4	9.0		
N	500			0.46			0.78	1.1	1.3	1.9	2.4	3.1	4.0	5.6			
P	800		0.29			0.49	0.67	0.84	1.2	1.5	1.9	2.5	3.5				
Q	1250	0.18			0.31	0.43	0.53	0.74	0.94	1.2	1.6	2.3					
R	2000			0.20	0.27	0.33	0.46	0.59	0.77	1.0	1.4						

TABLE VI-B—Limiting Quality (in defects per hundred units) for which P_a = 10 Percent (for Normal Inspection, Single sampling)

(See 11.6)

Code letter	Sample size	Acceptable Quality Level																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
A	2															120			200	270	330	460	590	770	1000	1400	1900
B	3														77			130	180	220	310	390	510	670	940	1300	1800
C	5													46			78	110	130	190	240	310	400	560	770	1100	
D	8												29			49	67	84	120	150	190	250	350	480	670		
E	13											18			30	41	51	71	91	120	160	220	300	410			
F	20										12			20	27	33	46	59	77	100	140						
G	32									7.2			12	17	21	29	37	48	63	88							
H	50								4.6			7.8	11	13	19	24	31	40	56								
J	80							2.9			4.9	6.7	8.4	12	15	19	25	35									
K	125						1.8			3.1	4.3	5.4	7.4	9.4	12	16	23										
L	200					1.2			2.0	2.7	3.3	4.6	5.9	7.7	10	14											
M	315				0.73			1.2	1.7	2.1	2.9	3.7	4.9	6.4	9.0												
N	500			0.46			0.78	1.1	1.3	1.9	2.4	3.1	4.0	5.6													
P	800		0.29			0.49	0.67	0.84	1.2	1.5	1.9	2.5	3.5														
Q	1250	0.18			0.31	0.43	0.53	0.74	0.94	1.2	1.6	2.3															
R	2000			0.20	0.27	0.33	0.46	0.59	0.77	1.0	1.4																

LQ (DEFECTS)
10%

LIMIT NUMBERS

TABLE VIII — Limit Numbers for Reduced Inspection

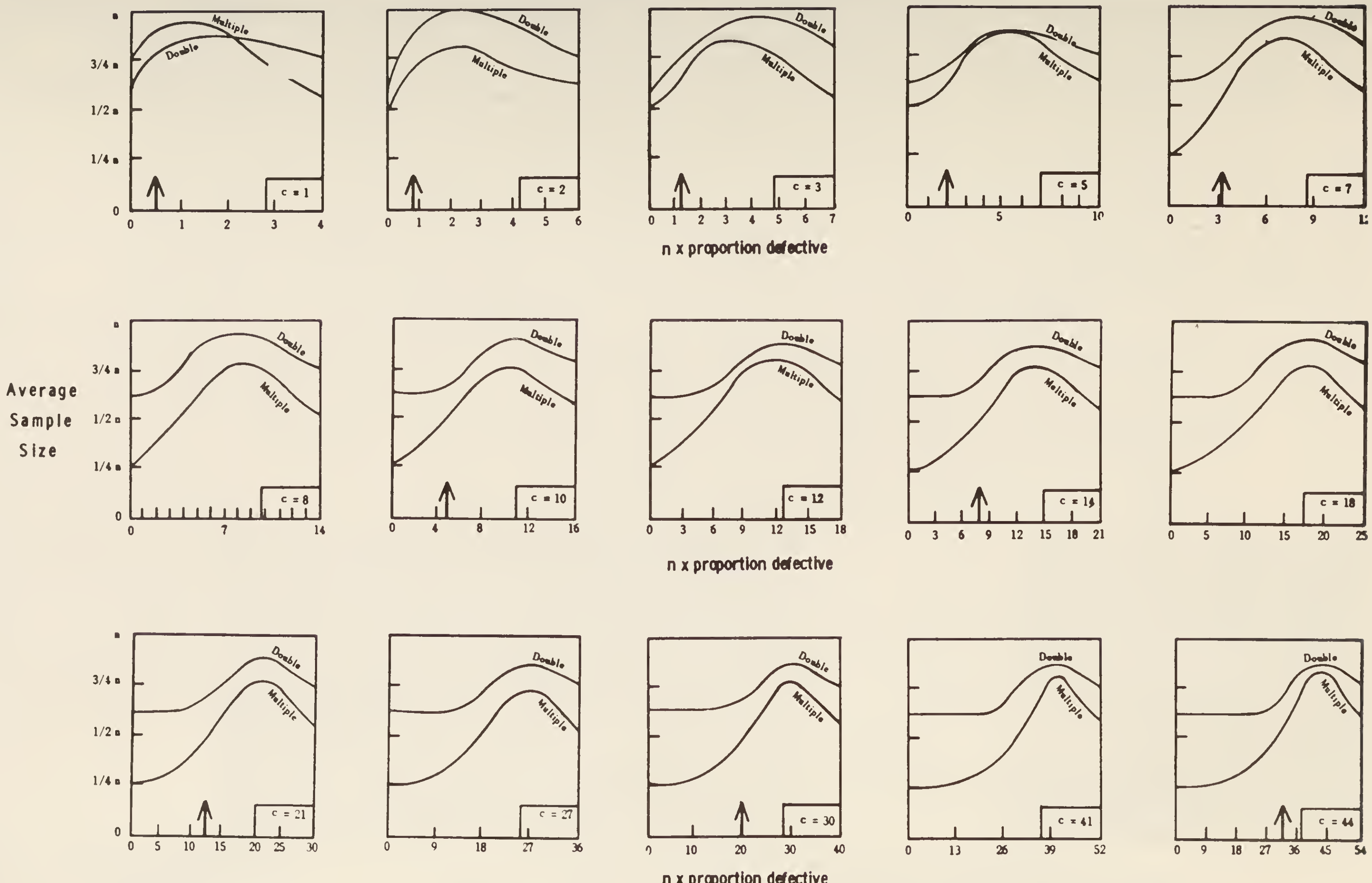
(See 8.3.3)

Number of sample units from last 10 lots or batches	Acceptable Quality Level																									
	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
20 - 29	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	0	0	2	4	8	14	22	40	68	115	181
30 - 49	•	•	•	•	•	•	•	•	•	•	•	•	•	•	0	0	1	3	7	13	22	36	63	105	178	277
50 - 79	•	•	•	•	•	•	•	•	•	•	•	•	•	0	0	2	3	7	14	25	40	63	110	181	301	
80 - 129	•	•	•	•	•	•	•	•	•	•	•	•	0	0	2	4	7	14	24	42	68	105	181	297		
130 - 199	•	•	•	•	•	•	•	•	•	•	•	0	0	2	4	7	13	25	42	72	115	177	301	490		
200 - 319	•	•	•	•	•	•	•	•	•	•	0	0	2	4	8	14	22	40	68	115	181	277	471			
320 - 499	•	•	•	•	•	•	•	•	•	0	0	1	4	8	14	24	39	68	113	189						
500 - 799	•	•	•	•	•	•	•	•	0	0	2	3	7	14	25	40	63	110	181							
800 - 1249	•	•	•	•	•	•	•	0	0	2	4	7	14	24	42	68	105	181								
1250 - 1999	•	•	•	•	•	•	0	0	2	4	7	13	24	40	69	110	169									
2000 - 3149	•	•	•	•	•	0	0	2	4	8	14	22	40	68	115	181										
3150 - 4999	•	•	•	•	0	0	1	4	8	14	24	38	67	111	186											
5000 - 7999	•	•	•	0	0	2	3	7	14	25	40	63	110	181												
8000 - 12499	•	•	0	0	2	4	7	14	24	42	68	105	181													
12500 - 19999	•	0	0	2	4	7	13	24	40	69	110	169														
20000 - 31499	0	0	2	4	8	14	22	40	68	115	181															
31500 - 49999	0	1	4	8	14	24	38	67	111	186																
50000 & Over	2	3	7	14	25	40	63	110	181	301																

Denotes that the number of sample units from the last ten lots or batches is not sufficient for reduced inspection for this AQL. In this instance more than ten lots or batches may be used for the calculation, provided that the lots or batches used are the most recent ones in sequence, that they have all been on normal inspection, and that none has been rejected while on original inspection.

TABLE IX — Average sample size curves for double and multiple sampling (normal and tightened inspection)

(See 11.5)



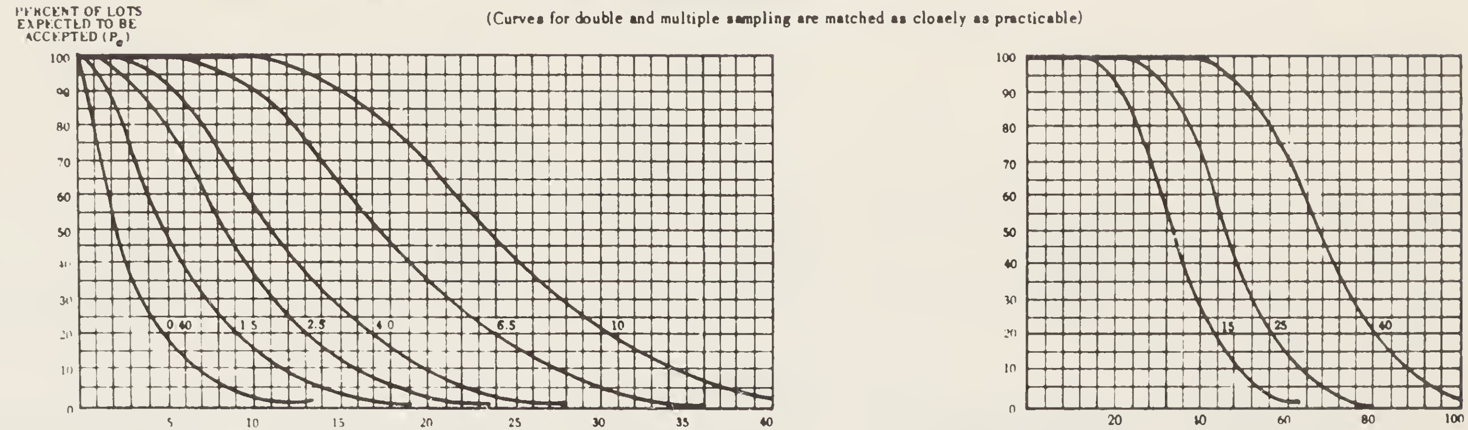
n = Equivalent single sample size
c = Single sample acceptance number
↑ = AQL for normal inspection

G

TABLE X-G—Tables for sample size code letter: G

CHART G - OPERATING CHARACTERISTIC CURVES FOR SINGLE SAMPLING PLANS

(Curves for double and multiple sampling are matched as closely as practicable)



QUALITY OF SUBMITTED LOTS (p, in percent defective for AQL's ≤ 10; in defects per hundred units for AQL's > 10)

Note: Figures on curves are Acceptable Quality Levels (AQL's) for normal inspection.

TABLE X-G-1 - TABULATED VALUES FOR OPERATING CHARACTERISTIC CURVES FOR SINGLE SAMPLING PLANS

P_a	Acceptable Quality Levels (normal inspection)																	
	0.40	1.5	2.5	4.0	6.5	10	0.40	1.5	2.5	4.0	6.5	10	X	15	X	25	X	40
	p (in percent defective)						p (in defects per hundred units)											
99.0	0.032	0.475	1.38	2.63	5.94	9.75	0.032	0.466	1.36	2.57	5.57	9.08	11.0	14.9	19.1	23.4	32.3	39.3
95.0	0.161	1.13	2.59	4.39	8.50	13.1	0.160	1.10	2.55	4.26	8.16	12.4	14.7	19.3	24.0	28.9	38.9	46.5
90.0	0.329	1.67	3.50	5.56	10.2	15.1	0.328	1.66	3.44	5.45	9.85	14.6	17.0	21.9	27.0	32.2	42.7	50.8
75.0	0.895	3.01	5.42	7.98	13.4	19.0	0.900	3.00	5.39	7.92	13.2	18.6	21.4	26.9	32.6	38.2	49.7	58.4
50.0	2.14	5.19	8.27	11.4	17.5	23.7	2.16	5.24	8.35	11.5	17.7	24.0	27.1	33.3	39.6	45.8	58.3	67.7
25.0	4.23	8.19	11.9	15.4	22.3	29.0	4.33	8.41	12.3	16.0	23.2	30.3	33.8	40.7	47.6	54.4	67.9	78.0
10.0	6.94	11.6	15.8	19.7	27.1	34.1	7.19	12.2	16.6	20.9	29.0	36.8	40.6	48.1	55.6	62.9	77.4	88.1
5.0	8.94	14.0	18.4	22.5	30.1	37.2	9.36	14.8	19.7	24.2	32.9	41.1	45.1	53.0	60.8	68.4	83.4	94.5
1.0	13.5	19.0	23.7	28.0	35.9	43.3	14.4	20.7	26.3	31.4	41.0	50.0	54.4	63.0	71.3	79.5	95.6	107
	0.65	2.5	4.0	6.5	10	X	0.65	2.5	4.0	6.5	10	X	15	X	25	X	40	X
	Acceptable Quality Levels (tightened inspection)																	

Note: Binomial distribution used for percent defective computations; Poisson for defects per hundred units

TABLE X-G-2 - SAMPLING PLANS FOR SAMPLE SIZE CODE LETTER: G

Type of sampling plan	Cumulative sample size	Less than 0.40	0.40	0.65	X	1.0	1.5	2.5	4.0	6.5	10	X	15	X	25	X	40	Higher than 40	Cumulative sample size
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	
Single	32	▽	0 1				1 2	2 3	3 4	5 6	7 8	8 9	10 11	12 13	14 15	18 19	21 22	△	32
Double	20	▽		Use	Use	Use	0 2	0 3	1 4	2 5	3 7	3 7	5 9	10	7 11	9 14	11 16	△	20
	40		*	Letter	Letter	Letter	1 2	3 4	4 5	6 7	8 9	11 12	12 13	15 16	18 19	23 24	26 27		40
Multiple	8	▽	*	F	J	H	# 2	# 2	# 3	# 4	0 4	0 4	0 5	0 6	1 7	1 8	2 9	△	8
	16						# 2	0 3	0 3	1 5	1 6	2 7	3 8	3 9	4 10	6 12	7 14		16
	24						0 2	0 3	1 4	2 6	3 8	4 9	6 10	7 12	8 13	11 17	13 19		24
	32						0 3	1 4	2 5	3 7	5 10	6 11	8 13	10 15	12 17	16 22	19 25		32
	40						1 3	2 4	3 6	5 8	7 11	9 12	11 15	14 17	17 20	22 25	25 29		40
	48						1 3	3 5	4 6	7 9	10 12	12 14	14 17	18 20	21 23	27 29	31 33		48
	56						2 3	4 5	6 7	9 10	13 14	14 15	18 19	21 22	25 26	32 33	37 38		56
Acceptable Quality Levels (tightened inspection)		Less than 0.65	0.65	X	1.0	1.5	2.5	4.0	6.5	10	X	15	X	25	X	40	X	Higher than 40	

Acceptable Quality Levels (normal inspection) head the upper columns; Acceptable Quality Levels (tightened inspection) are given in the bottom row.

△ = Use next preceding sample size code letter for which acceptance and rejection numbers are available.
▽ = Use next subsequent sample size code letter for which acceptance and rejection numbers are available.
Ac = Acceptance number.
Re = Rejection number.
* = Use single sampling plan above (or alternatively use letter K).
# = Acceptance not permitted at this sample size.

G

CHAPTER 5
CONTROL CHARTS

5.1 INTRODUCTION

Statistical quality control is designed to detect departures from product specifications and to indicate the need for remedial action. Acceptance sampling provides the basis for deciding whether to accept or reject a lot, whereas control charts are used to study the behaviorial pattern of ordered series of samples or subgroups.

A *control chart* is a graphical method of evaluating whether a process is or is not in a state of statistical control. A process is said to be in a statistical control if it is governed by random causes alone. Basic to the control chart is the recognition of two sources of variation: *random*, and *nonrandom*. Random variations are due to chance causes and are inherent in any process. Such variability cannot be altogether eliminated. The second type is variability associated with real changes in process level and are usually due to assignable causes that can be identified and eliminated. There are several advantages of a process being in a state of control, such as: product performance is predictable; corrective action can be taken quickly when there is evidence of deterioration; reduced variability ensures uniformity, inspection costs are low; and the decision to review/revise specifications is easier.

The concept and theory of control charts was developed by Dr. Walter A. Shewhart in 1924. The underlying idea was to apply the statistical principles of significance to the control of production processes. Many other types of control charts have been developed to date and the technique is being successfully applied to many quality control situations. For example, some of the areas where control charts can be used are: to evaluate performance of any activity or process; to identify possible causes of problems in a process; to identify peak performance periods of a process; to determine the repeatability of a measuring technique and its error; to decrease process variability; to reduce inspection level; to compare several inspectors, processes or laboratory analyses; etc. More specifically, it can be used for situations in which (i) one needs to establish a specification, (ii) review/revise a specification, and (iii) monitor a process.

5.2 NATURE OF CONTROL CHARTS

A control chart is a graph of the variability of the response variable. Samples of a given size are taken from various batches at more or less regular intervals and some statistic (mean, range, fraction defective, etc.) is computed and plotted for each sample. In addition, three horizontal lines are also plotted: a central line to indicate the overall average, an *upper control limit* (UCL), and a *lower control limit* (LCL). An outline of a control chart is shown in Figure 5.1.

At the basis of the control chart theory is Shewhart's central idea of division of observations into what are called *rational subgroups*. A rational subgroup is

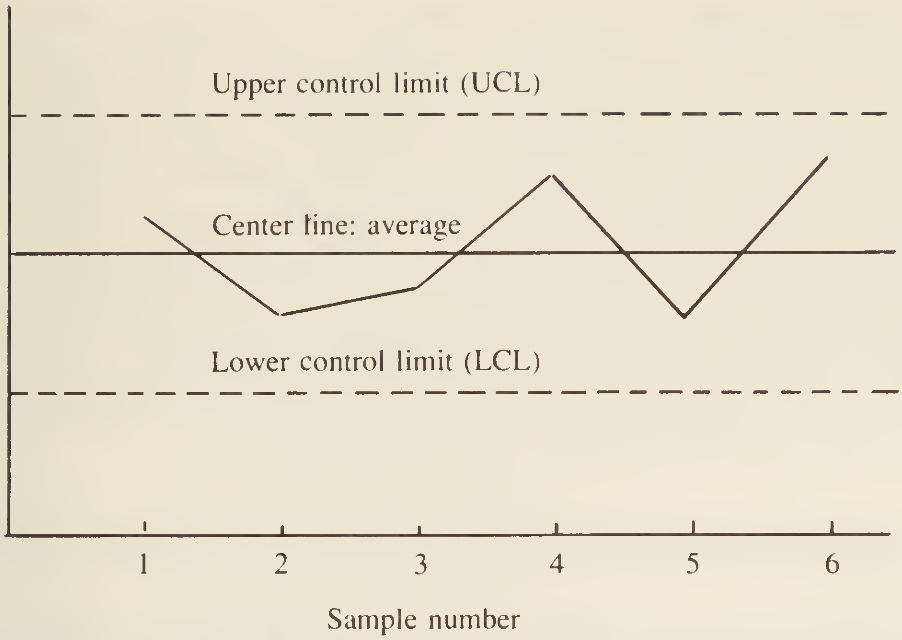


FIGURE 5.1 Outline of a control chart

one in which assignable causes are not likely to be present but which contains all the effects of sources of random error. Subgroups should be chosen in a way that is likely to give the maximum chance for the measurements in each subgroup to be alike and the maximum chance for the subgroups to differ one from the other. The mean of the subgroups, compared with each other, tell us how the general level of the process is changing; the standard deviation of the subgroups tell us how the within-group variability of the process is changing. The choice of the sample size n and the frequency of sampling depends on the process under consideration. Usually, the sample size is taken to be 4 to 6, while the sampling frequency is generally high in the beginning and low once a state of control is reached. Normally 20 to 25 samples of size 4 are considered adequate to provide good estimates.

Limits on the control charts are proposed by Shewhart at $3 - \sigma$ distance on each side of the central line, where σ is the population standard deviation. This means that 99.7% of the items in an operation will be included within the control limits, provided the operation is under control. It also means that there is a 0.3% chance, or three samples in a thousand, the upper or lower control limits may be violated. The possibility that this violation may occur should not be disregarded. When it occurs, action should be taken. Since action is required at this point we call this $3 - \sigma$ control limits the *action limits*. Further samples must be drawn to confirm whether or not the odd chance has occurred, or whether something has happened to the operation. If two further samples are drawn immediately, and both lie well within the $2 - \sigma$ limits, it can be assumed that an odd chance has occurred.

On the other hand, if the next two samples, while not outside the action limits, lie one just inside the $2 - \sigma$ limit and the other between the $2 - \sigma$ and the

action limit, the process should be investigated at once. The falling of any sample beyond the $2 - \sigma$ limit serves as a warning; therefore, the upper and lower $2 - \sigma$ control limits are called *warning limits*. If investigation indicates that more than 0.3% of the samples are outside the action limits, the process must be modified to avoid the defectives, or the specifications must be adjusted.

Note that a control chart can also have specification limits plotted on it. Specification limits characterize a product quality goal, whereas control limits are a measure of reality, that is, a measure of process quality capability. Control limits should always be within the specification limits for the process to be acceptable. However, it holds true for specification limits as well as control limits that if the limits are too wide the system will be too insensitive, and if the limits are too tight the system will be oversensitive.

5.3 TYPES AND CONSTRUCTION OF CONTROL CHARTS

Control charts are of two types: *variables* and *attributes*. Variable control charts apply to factors measured on a continuous scale (moisture, temperature, weight etc.) whereas attributes charts apply to factors that are counted (number of noxious weed seeds, number of dented cans, etc.) In Table 5.1. we give formulas for locating the central line and $3 - \sigma$ limit lines for the following Shewhart control charts:

1. *Variable control charts:* Mean: $\overline{X}$-chart; Range: R-chart; Standard deviation: σ-chart.
2. *Attributes control charts:* Fraction defective: p-chart; Number defective: np-chart; Number of defects: c-chart.

The formulas have been arranged according to whether or not standards are given, and whether R or S is to be used as a measure of subgroup variability. The values of the multiplying factors A, A_1, A_2, C_2, B_3, B_1, B_5, B_6, d_2, d_3, D_1, D_2, D_3, and D_4 are given in Appendix Table 7.

Control charts — standards given: The purpose here is to identify whether the observed values of $\overline{X}$, S, p, etc. for samples of n items differ from the respective standard values $\overline{X}_0$ (or μ), S_0 (or $d_2\,\sigma_0$), p_0, etc., by amounts greater than that expected to be due to chance causes only. The standard value can be a specification, an aimed-at value desired by management, a value dependent on economic considerations, or a value based on past experience. Control charts based on standards are used to help us decide whether or not the process is doing what we want it to do. The intent is to cause the plotted data to vary randomly about the aim.

Control charts — no standards given: Here a new process is being put into operation. No previous data is available, and no standards can be applied to this operation. The question is this: is this process in a state of statistical control, i.e. do data obtained from the process give us reason to believe that a constant cause system is at work?

TABLE 5.1 Control limit formulas for Shewhart control charts

	No standard given		Standard given	
Statistic	Central line	$3 - \sigma$ control limits	Central line	$3 - \sigma$ control limits
Variable control charts				
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_2\overline{R}$, $\overline{\overline{X}} \pm A_3\overline{S}$	$\overline{X}_0$ or μ	$\overline{X}_0 \pm A\sigma_0$
R	$\overline{R}$	$D_3\overline{R}$, $D_4\overline{R}$	R_0 or $d_2\sigma_0$	$D_1\sigma_0$, $D_2\sigma_0$
S	$\overline{S}$	$B_3\overline{S}$, $B_4\overline{S}$	S_0 or $c_4\sigma_0$	$B_5\sigma_0$, $B_6\sigma_0$
Attributes control charts				
p	$\overline{p}$	$\overline{p} \pm 3\sqrt{\overline{p}(1 - \overline{p})/n}$	p_0	$p_0 \pm 3\sqrt{p_0(1 - p_0)/n}$
np	$n\overline{p}$	$n\overline{p} \pm 3\sqrt{n\overline{p}(1 - \overline{p})}$	np_0	$np_0 \pm 3\sqrt{np_0(1 - p_0)}$
c	$\overline{c}$	$\overline{c} \pm 3\sqrt{\overline{c}}$	c_0	$c_0 \pm 3\sqrt{c_0}$

5.4 EXAMPLE: CONSTRUCTION OF $\overline{X}$, R, and σ CHARTS

Table 5.2. gives the average fill weight, in ounces, of 10 cans of tomato sauce taken at 6 different shift periods for 10 consecutive days at a cannery. The samples were taken for inspection to study the distribution of weights. We shall construct the control charts for the following two situations:

1. Assuming that no standard values are given and the inspection activity is being carried out to obtain information about the consistency of performance and variability of the quality characteristics.
2. The standard value being given, i.e., the desired values for the process control, were specified as $\overline{X}_0 = 19.5$ and $\sigma_0 = 0.05$.

TABLE 5.2 Average weight (ounces) of canned tomato sauce

Sample	1	2	3	4	5	6	Average $\overline{X}$	Range R	Standard deviation S
Day 1	17.9	19.5	19.3	19.2	18.2	19.1	18.87	1.6	0.653
Day 2	20.2	19.9	19.4	20.2	18.9	19.7	19.72	1.3	0.504
Day 3	20.0	19.0	19.3	20.7	19.4	19.1	19.1	1.7	0.649
Day 4	19.9	2.2	18.5	18.3	20.2	19.6	19.45	1.9	0.846
Day 5	19.7	20.4	19.0	21.0	20.3	19.7	20.02	2.0	0.677
Day 6	21.1	20.0	20.6	20.1	20.4	20.1	20.38	1.1	0.417
Day 7	19.3	20.3	20.5	20.2	20.4	20.0	2.012	1.2	0.436
Day 8	20.5	20.8	21.0	19.9	20.0	19.9	20.35	1.1	0.485
Day 9	20.8	20.0	20.5	20.1	20.7	20.6	20.45	0.8	0.327
Day 10	18.6	20.9	19.9	18.7	19.1	20.0	19.53	2.3	0.891
						Average	19.85	1.5	0.59

$\overline{\overline{X}} = 19.85$, $\overline{R} = 1.5$, and $\overline{S} = 0.59$

1. $\overline{X}$-Chart
   (i) Case 1: No standard given
   center line $= \overline{\overline{X}} = 19.85$
   $UCL_1 = \overline{\overline{X}} + A_3\overline{S} = 19.85 + 1.410 \times 0.59 = 20.68$
   and $LCL_1 = \overline{\overline{X}} - A_3\overline{S} = 19.85 - 1.410 \times 0.59 = 19.02$
   (ii) Case 2: Standard given ($\overline{\overline{X}} = 19.5$, $\sigma_0 = 0.5$)
   center line $= \overline{X}_0 = 19.5$
   $UCL_2 = \overline{X}_0 + A\sigma_0 = 19.5 + 1.225 \times 0.5 = 20.11$
   and $LCL_2 = \overline{X}_0 - A\sigma_0 = 19.5 - 1.225 \times 0.5 = 18.89$

NOTE: (UCL_1, LCL_1) represents control limits when no standard is given and (UCL_2, LCL_2) represents control limits when the standards are given. The charts are plotted on the same graph for both cases in Figure 5.2.

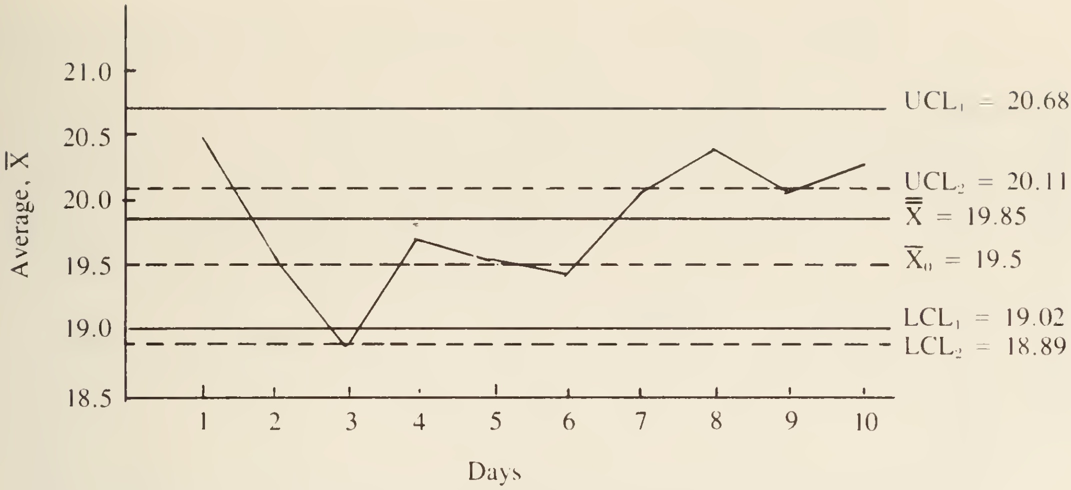


FIGURE 5.2 $\overline{X}$-chart for fill weight (ounces) of canned tomato sauce

2. R-Chart
   (i) Case 1: No standard given
   center line $= \overline{R} = 1.50$
   $UCL_1 = D_4\overline{R} = 2.004 \times 1.50 = 3.01$
   and $LCL_1 = D_3\overline{R} = 0 \times 1.50 = 0$
   (ii) Case 2: Standard given ($\sigma_0 = 0.5$)
   center line $= d_2\sigma_0 = 2.534 \times 0.5 = 1.27$
   $UCL_2 = D_2\sigma_0 = 5.078 \times 0.5 = 2.54$
   and $LCL_2 = D_1\sigma_0 = 0 \times 0.5 = 0$

See Figure 5.3.

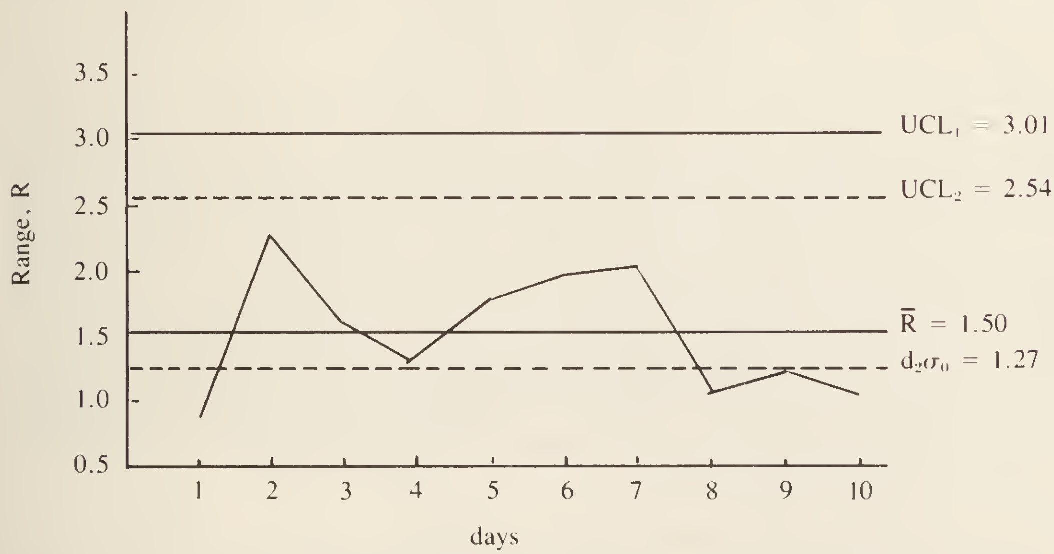


FIGURE 5.3 R-chart for fill weight (ounces) of canned tomato sauce

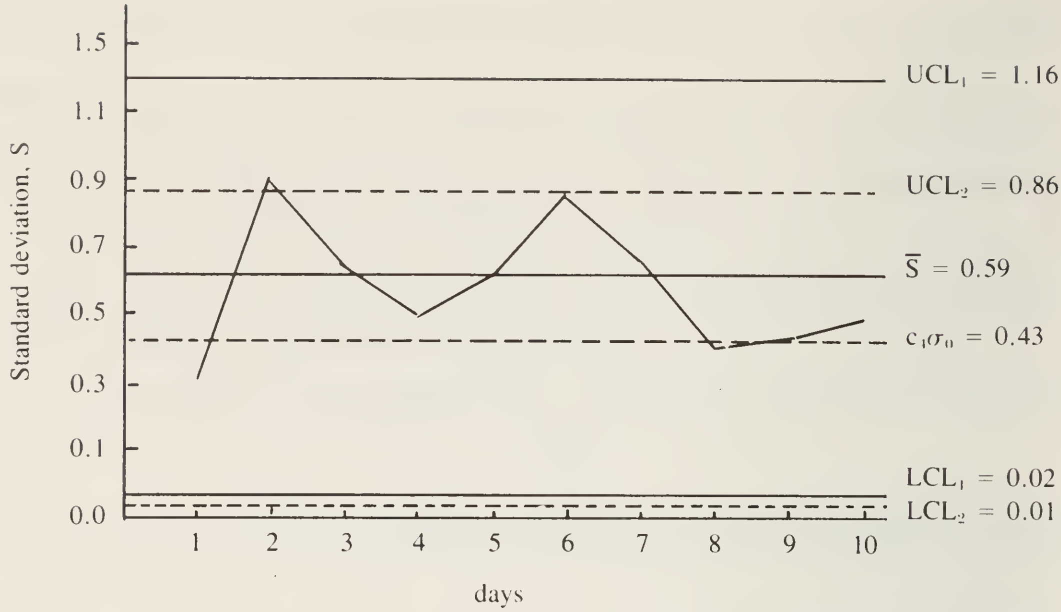


FIGURE 5.4 σ-chart for fill weight (ounces) of canned tomato sauce

3. σ-Chart
   (i) Case 1: No standard given
   center line $= \overline{S} = 0.59$
   $UCL_1 = B_4\overline{S} = 1.970 \times 0.59 = 1.16$
   and $LCL_1 = B_3\overline{S} = 0.03 \times 0.59 = 0.02$
   (ii) Case 2: Standard given
   center line $= c_4\sigma_0 = 0.8686 \times 0.5 = 0.43$
   $UCL_2 = B_6\sigma_0 = 1.711 \times 0.5 = 0.86$
   and $LCL_2 = B_5\sigma_0 = 0.026 \times 0.5 = 0.01$

See Figure 5.4.

RESULTS: In most cannery operations, the problem is to control the filling process within certain tolerances with a specific point or figure being the target. A chart tells at a glance if variations in the operation are significant or if they are just minor due to normal chance causes. The chart can also show trends or wild and unpredictable fluctuations that could require readjustment of the filler.

When no standards are given, the aim is to obtain information on performance consistency and variability pattern of the process. This also tells us when to leave the process alone and when to take action. On the other hand, one may like to specify a standard value and expect to reduce wastage due to overfilling or rejections due to underfilling.

Here, the $\overline{X}$-chart indicates a tendency of the process toward overfilling in terms of the given standard values. The process seems to be under control most of the days except on Day 3. This "out of control" point indicates something was wrong at about that time and that the variations were not all due to chance, therefore calling for immediate action. In case of the R-chart, only the upper control limit is marked, below which chance variations occur. No lower limit is shown because it would be zero or calculate to less than zero, and one cannot have less than zero weight.

5.5 EXAMPLE: CONSTRUCTION OF p-CHART

A study was made to estimate the efficiency of a labeling machine putting labels on a canned food product. Samples of 50 cans were taken from each day's output and visually inspected to find out the number of cans wrongly labeled. The total number of wrongly labeled cans was counted for each day's output for 8 days. The data obtained are given in Table 5.3.

TABLE 5.3 Improperly labeled cans from example 5.5

Day	1	2	3	4	5	6	7	8	Total
No. of cans wrongly labeled	3	4	12	14	2	4	3	1	43

$$\text{Mean fraction defective} = \bar{p} = \frac{43}{50 \times 8} = 0.11$$

$$\text{Estimate of the standard deviation } (\sigma_p) = S_p = \sqrt{\frac{\bar{p}\,(1 - \bar{p})}{n}}$$

$$= \sqrt{\frac{0.11\,(1 - 0.11)}{50}} = 0.044$$

Thus, for the p-chart,

$$\text{center line} = \bar{p} = 0.11$$
$$\text{UCL} = \bar{p} + 3S_p = 0.11 + 3 \times 0.044 = 0.242$$
$$\text{LCL} = \bar{p} - 3S_p = 0.11 - 3 \times 0.044$$
$$= 0.0 \text{ (since LCL} < 0.0)$$

Figure 5.5 shows the control chart and associated points for the 8 days of inspection.

At day 3, the machine is beginning to go out of control and by 4 it is completely out of control. The cause of the problem was found and rectified and the machine stayed in control afterwards.

To establish the final control limits that can be used for future studies of the process/machine capability, these p-chart limits are reset. The two points corresponding to days 3 and 4 are removed from the analysis and the control limits are recalculated as follows:

$$\Sigma_p = 17$$

$$\bar{p} = \frac{17}{6 \times 50} = 0.057$$

$$S_p = \sqrt{\frac{0.057\,(1 - 0.057)}{50}} = 0.033$$

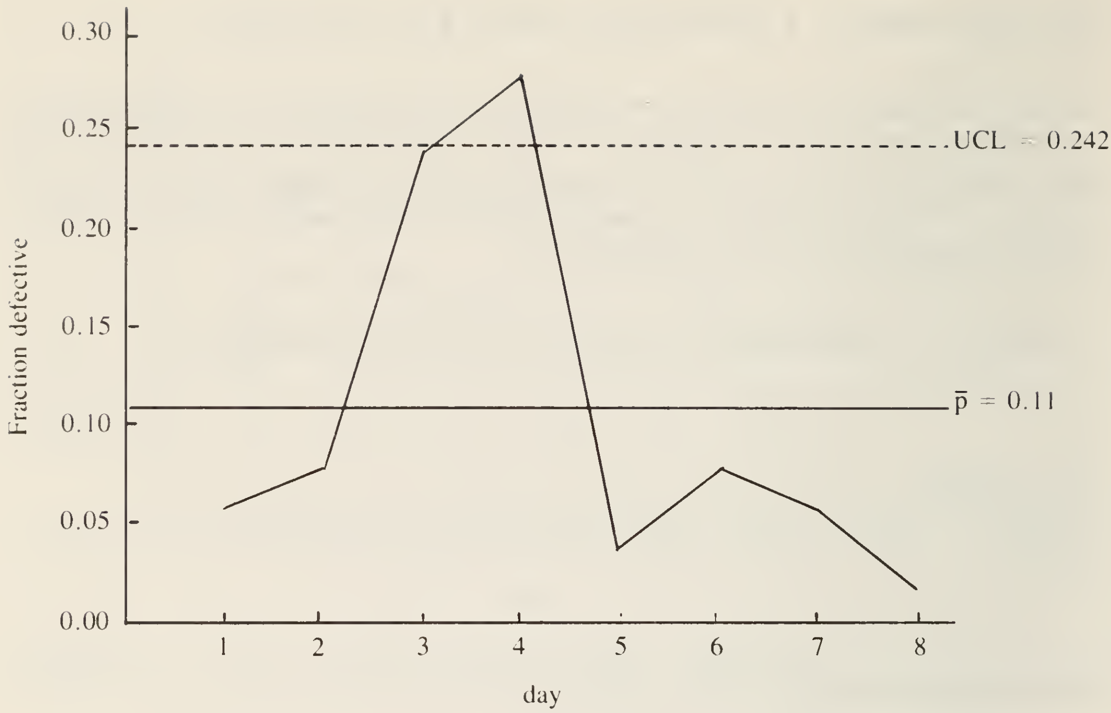


FIGURE 5.5 p-chart for labeling operation

Thus, for the p-chart,

$$\begin{aligned} \text{center line} &= \bar{p} = 0.057 \\ \text{UCL} &= \bar{p} + 3S_p = 0.057 + 3 \times 0.033 = 0.155 \\ \text{LCL} &= \bar{p} - 3S_p = 0.057 - 3 \times 0.033 \\ &= 0.0 \text{ (since LCL} < 0.0) \end{aligned}$$

Figure 5.6 gives the new control limits with the data from 6 days (days 1, 2, 5, 6, 7, 8). As can be seen, these data are randomly distributed around the center line well within the control limits, exhibiting a state of statistical control for the machine.

5.6 REMARKS ON $\overline{X}$-, R- and p-CHARTS

The $\overline{X}$-chart shows where the process is centered, and is an indicator of the stability of the process. In some cases it also indicates the relationship between the process and the specification. The $\overline{X}$-chart reveals undesirable variations between samples as far as their average is concerned, while the R-chart reveals any undesirable variation within samples.

The R-chart is an indicator of the magnitude of the variability of the process being studied. It is a measure of process consistency or uniformity. It is desirable to have a level on an R-chart as low as possible. The R-chart stays in control if the variations within samples are the same. This happens only if all the samples receive the same treatment. If the R-chart does not remain in control, or if its

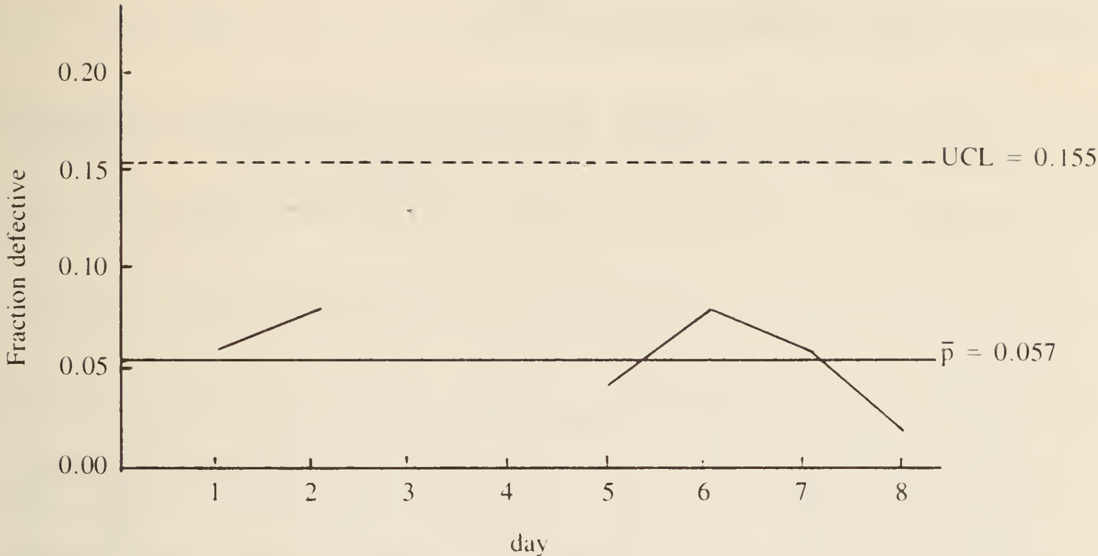


FIGURE 5.6 Revised p-chart for labeling operation

level rises, it indicates that either different samples are getting different treatment or several different cause-effect systems are operating on the process.

The p-chart is used to determine the average percent of defective items submitted over a period of time. It brings to the attention of management any changes in this average. The process is judged to be in statistical control in the same way as is done for $\overline{X}$- and R-charts. If all the sample points fall within the control limits without exhibiting any specific pattern, the process is said to be in control. In such a case the average fraction defective, $\overline{p}$, is taken as the standard fraction defective. The cost of collecting data for p-charts is generally less than that required for $\overline{X}$- and R-charts.

Since the main objective of control charts is to identify when a process is out of control, we give below some situations that indicate lack of process control:

1. A point going outside the control limits is an indication of the presence of assignable causes.
2. A run of seven or more points above or below the central line produces a pattern and indicates a shift in the process level. On the R-chart a run of points above the central line is indicative of an increase in process spread, which represents an undesirable situation. A run of points below the central line is also significant because it indicates a definite improvement in the sense of variability and, therefore, identifies a need to consider revision of tolerances.
3. Sometimes a run of points beyond some secondary limits, e.g., a run of two to three points beyond $2 - \sigma$ limits, or run of four to five points beyond $1 - \sigma$ limits, or even a point in the vicinity of the control limits, is taken to be a warning of a possible lack of control or presence of nonrandom causes.

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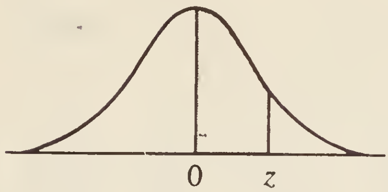
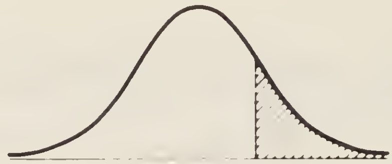
APPENDIX

TABLE 1 Areas under the normal probability curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 2 Percentage points of the t-distribution

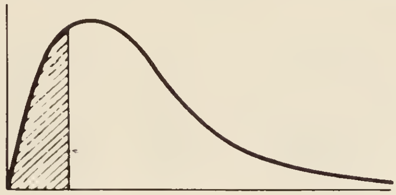


t_{α}

d f	$t_{.40}$	$t_{.30}$	$t_{.20}$	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.0005}$
1	0 3250	0 7270	1 376	3 078	6 3138	12 706	31 821	63 657	636 619
2	2885	6172	1 061	1 886	2 9200	4 3027	6 965	9 9248	31 598
3	2766	5840	978	1 638	2 3534	3 1825	4 541	5 8409	12 924
4	2707	5692	941	1.533	2 1318	2 7764	3 747	4 6041	8 610
5	2672	.5598	.920	1 476	2 0150	2 5706	3 365	4 0321	6 869
6	2648	5536	.906	1 440	1 9432	2 4469	3 143	3 7074	5 959
7	2632	5493	896	1 415	1 8946	2 3646	2 998	3 4995	5 408
8	2619	5461	889	1 397	1 8595	2 3060	2 896	3 3554	5.041
9	2610	5436	.883	1 383	1 8331	2 2622	2 821	3 2498	4 781
10	2602	5416	.879	1 372	1 8125	2 2281	2 764	3 1693	4 587
11	2596	5400	876	1 363	1 7939	2 2010	2 718	3 1058	4 437
12	2590	5387	873	1 356	1 7823	2 1788	2 681	3 0545	4 318
13	2586	5375	870	1 350	1 7709	2 1604	2 650	3 0123	4 221
14	2582	5366	868	1 345	1 7613	2 1448	2 624	2 9768	4 140
15	2579	5358	.866	1 341	1 7530	2 1315	2 602	2 9467	4 073
16	2576	5351	.865	1 337	1 7459	2 1199	2 583	2 9208	4 015
17	.2574	5344	.863	1 333	1 7396	2 1098	2 567	2 8982	3 965
18	2571	5338	.862	1 330	1 7341	2 1009	2 552	2 8784	3 922
19	2569	5333	.861	1 328	1 7291	2 0930	2 539	2 8609	3 883
20	2567	5329	.860	1.325	1 7247	2 0860	2.528	2 8453	3 850
21	2566	5325	.859	1 323	1 7207	2 0796	2.518	2 8314	3 819
22	2564	5321	858	1 321	1.7171	2 0739	2.508	2 8188	3 792
23	2563	5318	858	1 319	1 7139	2 0687	2.500	2 9073	3 767
24	2562	5315	857	1 318	1 7109	2 0639	2 492	2 7969	3 745
25	2561	5312	856	1 316	1 7081	2 0595	2 485	2 7874	3 725
26	2560	5309	.856	1 315	1 7056	2 0555	2 479	2 7787	3 707
27	.2559	5307	855	1 314	1 7033	2 0518	2 473	2 7707	3 690
28	2558	.5304	855	1 313	1 7011	2 0484	2 467	2 7633	3 674
29	2557	.5302	.854	1 311	1 6991	2 0452	2 462	2 7564	3 659
30	2556	5300	.854	1 310	1 6973	2 0423	2 457	2 7500	3 616
35	2553	5292	8521	1.3062	1 6896	2 0301	2 438	2 7239	3 5919
40	2550	5286	8507	1 3031	1 6839	2 0211	2 423	2 7045	3 5511
45	2549	5281	8497	1 3007	1 6794	2 0141	2 412	2 6896	3 5207
50	2547	5278	8489	1 2987	1 6759	2 0086	2 403	2 6778	3 4965
60	.2545	.5272	8477	1 2959	1 6707	2 0003	2 390	2 6603	3 4606
70	2543	5268	8468	1 2938	1.6669	1 9945	2 381	2 6480	3 4355
80	2542	5265	8462	1 2922	1 6641	1 9901	2 374	2 6388	3 4169
90	2541	5263	8457	1 2910	1 6620	1 9867	2.368	2 6316	3 4022
100	2540	5261	8452	1 2901	1 6602	1.9840	2 364	2 6260	3 3909
120	2539	5258	8446	1 2887	1 6577	1 9799	2 358	2 6175	3 3736
140	2538	5256	8442	1 2876	1 6558	1 9771	2 353	2 6114	3 3615
160	2538	5255	8439	1 2869	1 6545	1 9749	2 350	2 6070	3 3527
180	2537	.5253	8436	1 2863	1 6534	1.9733	2 347	2 6035	3 3456
200	2537	5252	8434	1 2858	1 6525	1 9719	2 345	2 6006	3 3400
∞	2533	5244	8416	1 2816	1 6449	1 9600	2 326	2 5758	3 2905

SOURCE: Reproduced from *Documenta Geigy Scientific Tables*, 7th edition, by permission of CIBA-GEIGY Limited, Basle, Switzerland.

TABLE 3 Percentage points of the chi-square distribution



d.f.	$\chi^2_{.005}$	$\chi^2_{.010}$	$\chi^2_{.025}$	$\chi^2_{.050}$	$\chi^2_{.100}$	$\chi^2_{.250}$	$\chi^2_{.500}$
1	392704.10^{-10}	157088.10^{-9}	982069.10^{-9}	393214.10^{-8}	0·0157908	0·1015308	0·454936
2	0·0100251	0·0201007	0·0506356	0·102587	0·210721	0·575364	1·38629
3	0·0717218	0·114832	0·215795	0·351846	0·584374	1·212534	2·36597
4	0·206989	0·297109	0·484419	0·710723	1·063623	1·92256	3·35669
5	0·411742	0·554298	0·831212	1·145476	1·61031	2·67460	4·35146
6	0·675727	0·872090	1·23734	1·63538	2·20413	3·45460	5·34812
7	0·989256	1·239043	1·68987	2·16735	2·83311	4·25485	6·34581
8	1·34441	1·64650	2·17973	2·73264	3·48954	5·07064	7·34412
9	1·73493	2·08790	2·70039	3·32511	4·16816	5·89883	8·34283
10	2·15586	2·55821	3·24697	3·94030	4·86518	6·73720	9·34182
11	2·60322	3·05348	3·81575	4·57481	5·57778	7·58414	10·3410
12	3·07382	3·57057	4·40379	5·22603	6·30380	8·43842	11·3403
13	3·56503	4·10692	5·00875	5·89186	7·04150	9·29907	12·3398
14	4·07467	4·66043	5·62873	6·57063	7·78953	10·1653	13·3393
15	4·60092	5·22935	6·26214	7·26094	8·54676	11·0365	14·3389
16	5·14221	5·81221	6·90766	7·96165	9·31224	11·9122	15·3385
17	5·69722	6·40776	7·56419	8·67176	10·0852	12·7919	16·3382
18	6·26480	7·01491	8·23075	9·39046	10·8649	13·6753	17·3379
19	6·84397	7·63273	8·90652	10·1170	11·6509	14·5620	18·3377
20	7·43384	8·26040	9·59078	10·8508	12·4426	15·4518	19·3374
21	8·03365	8·89720	10·28293	11·5913	13·2396	16·3444	20·3372
22	8·64272	9·54249	10·9823	12·3380	14·0415	17·2396	21·3370
23	9·26043	10·19567	11·6886	13·0905	14·8480	18·1373	22·3369
24	9·88623	10·8564	12·4012	13·8484	15·6587	19·0373	23·3367
25	10·5197	11·5240	13·1197	14·6114	16·4734	19·9393	24·3366
26	11·1602	12·1981	13·8439	15·3792	17·2919	20·8434	25·3365
27	11·8076	12·8785	14·5734	16·1514	18·1139	21·7494	26·3363
28	12·4613	13·5647	15·3079	16·9279	18·9392	22·6572	27·3362
29	13·1211	14·2565	16·0471	17·7084	19·7677	23·5666	28·3361
30	13·7867	14·9535	16·7908	18·4927	20·5992	24·4776	29·3360
40	20·7065	22·1643	24·4330	26·5093	29·0505	33·6603	39·3353
50	27·9907	29·7067	32·3574	34·7643	37·6886	42·9421	49·3349
60	35·5345	37·4849	40·4817	43·1880	46·4589	52·2938	59·3347
70	43·2752	45·4417	48·7576	51·7393	55·3289	61·6983	69·3345
80	51·1719	53·5401	57·1532	60·3915	64·2778	71·1445	79·3343
90	59·1963	61·7541	65·6466	69·1260	73·2911	80·6247	89·3342
100	67·3276	70·0649	74·2219	77·9295	82·3581	90·1332	99·3341
X	−2·5758	−2·3263	−1·9600	−1·6449	−1·2816	−0·6745	0·0000

TABLE 3 (Continued)

d.f.	$\chi^2_{.750}$	$\chi^2_{.900}$	$\chi^2_{.950}$	$\chi^2_{.975}$	$\chi^2_{.990}$	$\chi^2_{.995}$	$\chi^2_{.999}$
1	1·32330	2·70554	3·84146	5·02389	6·63490	7·87944	10·828
2	2·77259	4·60517	5·99146	7·37776	9·21034	10·5966	13·816
3	4·10834	6·25139	7·81473	9·34840	11·3449	12·8382	16·266
4	5·38527	7·77944	9·48773	11·1433	13·2767	14·8603	18·467
5	6·62568	9·23636	11·0705	12·8325	15·0863	16·7496	20·515
6	7·84080	10·6446	12·5916	14·4494	16·8119	18·5476	22·458
7	9·03715	12·0170	14·0671	16·0128	18·4753	20·2777	24·322
8	10·2189	13·3616	15·5073	17·5345	20·0902	21·9550	26·125
9	11·3888	14·6837	16·9190	19·0228	21·6660	23·5894	27·877
10	12·5489	15·9872	18·3070	20·4832	23·2093	25·1882	29·588
11	13·7007	17·2750	19·6751	21·9200	24·7250	26·7568	31·264
12	14·8454	18·5493	21·0261	23·3367	26·2170	28·2995	32·909
13	15·9839	19·8119	22·3620	24·7356	27·6882	29·8195	34·528
14	17·1169	21·0641	23·6848	26·1189	29·1412	31·3194	36·123
15	18·2451	22·3071	24·9958	27·4884	30·5779	32·8013	37·697
16	19·3689	23·5418	26·2962	28·8454	31·9999	34·2672	39·252
17	20·4887	24·7690	27·5871	30·1910	33·4087	35·7185	40·790
18	21·6049	25·9894	28·8693	31·5264	34·8053	37·1565	42·312
19	22·7178	27·2036	30·1435	32·8523	36·1909	38·5823	43·820
20	23·8277	28·4120	31·4104	34·1696	37·5662	39·9968	45·315
21	24·9348	29·6151	32·6706	35·4789	38·9322	41·4011	46·797
22	26·0393	30·8133	33·9244	36·7807	40·2894	42·7957	48·268
23	27·1413	32·0069	35·1725	38·0756	41·6384	44·1813	49·728
24	28·2412	33·1962	36·4150	39·3641	42·9798	45·5585	51·179
25	29·3389	34·3816	37·6525	40·6465	44·3141	46·9279	52·618
26	30·4346	35·5632	38·8851	41·9232	45·6417	48·2899	54·052
27	31·5284	36·7412	40·1133	43·1945	46·9629	49·6449	55·476
28	32·6205	37·9159	41·3371	44·4608	48·2782	50·9934	56·892
29	33·7109	39·0875	42·5570	45·7223	49·5879	52·3356	58·301
30	34·7997	40·2560	43·7730	46·9792	50·8922	53·6720	59·703
40	45·6160	51·8051	55·7585	59·3417	63·6907	66·7660	73·402
50	56·3336	63·1671	67·5048	71·4202	76·1539	79·4900	86·661
60	66·9815	74·3970	79·0819	83·2977	88·3794	91·9517	99·607
70	77·5767	85·5270	90·5312	95·0232	100·425	104·215	112·317
80	88·1303	96·5782	101·879	106·629	112·329	116·321	124·839
90	98·6499	107·565	113·145	118·136	124·116	128·299	137·208
100	109·141	118·498	124·342	129·561	135·807	140·169	149·449
X	+ 0·6745	+ 1·2816	+ 1·6449	+ 1·9600	+ 2·3263	+ 2·5758	+ 3·0902

SOURCE: Abridged with permission from *Biometrika Tables for Statisticians*, Vol. 1. Edited by E. S. Pearson and H. O. Hartley, Cambridge University Press (1966).

TABLE 4 95% Two-sided tolerance interval: μ, σ unknown
$\overline{X} \pm k_2'(n, p) S$: values of the coefficient $k_2'(n, p)$

n	p = 0.90	p = 0.95	p = 0.99
5	4.28	5.08	6.63
6	3.71	4.41	5.78
7	3.37	4.01	5.25
8	3.14	3.73	4.89
9	2.97	3.53	4.63
10	2.84	3.38	4.43
11	2.74	3.26	4.28
12	2.66	3.16	4.15
13	2.59	3.08	4.04
14	2.53	3.01	3.96
15	2.48	2.95	3.88
16	2.44	2.90	3.81
17	2.40	2.86	3.75
18	2.37	2.82	3.70
19	2.34	2.78	3.66
20	2.31	2.75	3.62
22	2.26	2.70	3.54
24	2.23	2.65	3.48
26	2.19	2.61	3.43
28	2.16	2.58	3.39
30	2.14	2.55	3.35
35	2.09	2.49	3.27
40	2.05	2.45	3.21
45	2.02	2.41	3.17
50	2.00	2.38	3.13
60	1.96	2.33	3.07
70	1.93	2.30	3.02
80	1.91	2.27	2.99
90	1.89	2.25	2.96
100	1.87	2.23	2.93
150	1.83	2.18	2.86
200	1.80	2.14	2.82
250	1.78	2.12	2.79
300	1.77	2.11	2.77
400	1.75	2.08	2.74
500	1.74	2.07	2.72
1000	1.71	2.04	2.68
∞	1.64	1.96	2.58

SOURCE: Abridged with permission from *ISO International Standard 3207*. Complete document available from ISO Central Secretariate, Geneva, or Standards Council of Canada.

TABLE 5 Random numbers

93108	77033	68325	10160	38667	62441	87023	94372	06164	30700
28271	08589	83279	48838	60935	70541	53814	95588	05832	80235
21841	35545	11148	34775	17308	88034	97765	35959	52843	44895
22025	79554	19698	25255	50283	94037	57463	92925	12042	91414
09210	20779	02994	02258	86978	85092	54052	18354	20914	28460
90552	71129	03621	20517	16908	06668	29916	51537	93658	29525
01130	06995	20258	10351	99248	51660	38861	49668	74742	47181
22604	56719	21784	68788	38358	59827	19270	99287	81193	43366
06690	01800	34272	65497	94891	14537	91358	21587	95765	72605
59809	69982	71809	64984	48709	43991	24987	69246	86400	29559
56475	02726	58511	95405	70293	84971	06676	44075	32338	31980
02730	34870	83209	03138	07715	31557	55242	61308	26507	06186
74482	33990	13509	92588	10462	76546	46097	01825	20153	36271
19793	22487	94238	81054	95488	23617	15539	94335	73822	93481
19020	27856	60526	24144	98021	60564	46373	86928	52135	74919
69565	60635	65709	77887	42766	86698	14004	94577	27936	47220
69274	23208	61035	84263	15034	28717	76146	22021	23779	98562
83658	14204	09445	41081	49630	34215	89806	40930	97194	21747
78612	51102	66826	40430	54072	62164	68977	95583	11765	81072
14980	74158	78216	38985	60838	82836	42777	85321	90463	11813
63172	28010	29405	91554	75195	51183	65805	87525	35952	83204
71167	37984	52737	06869	38122	95322	41356	19391	96787	64410
78530	56410	19195	34434	83712	50397	80920	15464	81350	18673
98324	03774	07573	67864	06497	20758	83454	22756	83959	96347
55793	30055	08373	32652	02654	75980	02095	87545	88815	80086
05674	34471	61967	91266	38814	44728	32455	17057	08339	93997
15643	22245	07592	22078	73628	60902	41561	54608	41023	98345
66750	19609	70358	03622	64898	82220	69304	46235	97332	64539
42320	74314	50222	82339	51564	42885	50482	98501	02245	88990
73752	73818	15470	04914	24936	65514	56633	72030	30856	85183
97546	02188	46373	21486	28221	08155	23486	66134	88799	49496
32569	52162	38444	42004	78011	16909	94194	79732	47114	23919
36048	93973	82596	28739	86985	58144	65007	08786	14826	04896
40455	36702	38965	56042	80023	28169	04174	65533	52718	55255
33597	47071	55618	51796	71027	46690	08002	45066	02870	60012
22828	96380	35883	15910	17211	42358	14056	55438	98148	35384
00631	95925	19324	31497	88118	06283	84596	72091	53987	01477
75722	36478	07634	63114	27164	15467	03983	09141	60562	65725
80577	01771	61510	17099	28731	41426	18853	41523	14914	76661
10524	20900	65463	83680	05005	11611	64426	59065	06758	02892
93815	69446	75253	51915	97839	75427	90685	60352	96288	34248
81867	97119	93446	20862	46591	97677	42704	13718	44975	67145
64649	07689	16711	12169	15238	74106	60655	56289	74166	78561
55768	09210	52439	33355	57884	36791	00853	49969	74814	09270
38080	49460	48137	61589	42742	92035	21766	19435	92579	27683
22360	16332	05343	34613	24013	98831	17157	44089	07366	66196
40521	09057	00239	51284	71556	22605	41293	54854	39736	05113
19292	69862	59951	49644	53486	28244	20714	56030	39292	45166
79504	40078	06838	05509	68581	39400	85615	52314	83202	40313
64138	27983	84048	42631	58658	62243	82572	45211	37060	15017

SOURCE: Abstracted with permission from *A Million Random Digits with 100,000 Normal Deviates*, The Rand Corporation, Santa Maria, Calif.

TABLE 6 Summation of terms of poisson's exponential binomial limit

	c—Acceptable Number of Defects																
np'	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.02	980	1,000															
0.04	961	999	1,000														
0.06	942	998	1,000														
0.08	923	997	1,000														
0.10	905	995	1,000														
0.15	861	990	999	1,000													
0.20	819	982	999	1,000													
0.25	779	974	998	1,000													
0.30	741	963	996	1,000													
0.35	705	951	994	1,000													
0.40	670	938	992	999	1,000												
0.45	638	925	989	999	1,000												
0.50	607	910	986	998	1,000												
0.55	577	894	982	998	1,000												
0.60	549	878	977	997	1,000												
0.65	522	861	972	996	999	1,000											
0.70	497	841	966	994	999	1,000											
0.75	472	827	959	993	999	1,000											
0.80	449	809	953	991	999	1,000											
0.85	427	791	945	989	998	1,000											
0.90	407	772	937	987	998	1,000											
0.95	387	754	929	984	997	1,000											
1.00	368	736	920	981	996	999	1,000										
1.1	333	699	900	974	995	999	1,000										
1.2	301	663	879	966	992	998	1,000										
1.3	273	627	857	957	989	998	1,000										
1.4	247	592	833	946	986	997	999	1,000									
1.5	223	558	809	934	981	996	999	1,000									
1.6	202	525	783	921	976	994	999	1,000									
1.7	183	493	757	907	970	992	998	1,000									
1.8	165	463	731	891	964	990	997	999	1,000								
1.9	150	434	704	875	956	987	997	999	1,000								
2.0	135	406	677	857	947	983	995	999	1,000								
2.2	111	355	623	819	928	975	993	998	1,000								
2.4	091	308	570	779	904	964	988	997	999	1,000							
2.6	074	267	518	736	877	951	983	995	999	1,000							
2.8	061	231	469	692	848	935	976	992	998	999	1,000						
3.0	050	199	423	647	815	916	966	988	996	999	1,000						
3.2	041	171	380	603	781	895	955	983	994	998	1,000						
3.4	033	147	340	558	744	871	942	977	992	997	999	1,000					
3.6	027	126	303	515	706	844	927	969	988	996	999	1,000					
3.8	022	107	269	473	668	816	909	960	984	994	998	999	1,000				

np' = number of samples (n) multiplied by the defect fraction p'.

TABLE 6 (Continued)

np'	c— Acceptable Number of Defects																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4.0	018	092	238	433	629	785	889	949	979	992	997	999	1,000				
4.2	015	078	210	395	590	753	867	936	972	989	996	999	1,000				
4.4	012	066	185	359	551	720	844	921	964	985	994	998	999	1,000			
4.6	010	056	163	326	513	686	818	905	955	980	992	997	999	1,000			
4.8	008	048	143	294	476	651	791	887	944	975	990	996	999	1,000			
5.0	007	040	125	265	440	616	762	867	932	968	986	995	998	999	1,000		
5.2	006	034	109	238	406	581	732	845	918	960	982	993	997	999	1,000		
5.4	005	029	095	213	373	546	702	822	903	951	977	990	996	999	1,000		
5.6	004	024	082	191	342	512	670	797	886	941	972	988	995	998	999	1,000	
5.8	003	021	072	170	313	478	638	771	867	929	965	984	993	997	999	1,000	
6.0	002	017	062	151	285	446	606	744	847	916	957	980	991	996	999	999	1,000
6.2	002	015	054	134	259	414	574	716	826	902	949	975	989	995	998	999	1,000
6.4	002	012	046	119	235	384	542	687	803	886	939	969	986	994	997	999	1,000
6.6	001	010	040	105	213	355	511	658	780	869	927	963	982	992	997	999	999
6.8	001	009	034	093	192	327	480	628	755	850	915	955	978	990	996	998	999
7.0	001	007	030	082	173	301	450	599	729	830	901	947	973	987	994	998	999
7.2	001	006	025	072	156	276	420	569	703	810	887	937	967	984	993	997	999
7.4	001	005	022	063	140	253	392	539	676	788	871	926	961	980	991	996	998
7.6	001	004	019	055	125	231	365	510	648	765	854	915	954	976	989	995	998
7.8	000	004	016	048	112	210	338	481	620	741	835	902	945	971	986	993	997
8.0	000	003	014	042	100	191	313	453	593	717	816	888	936	966	983	992	996
8.5	000	002	009	030	074	150	256	386	523	653	763	849	909	949	973	986	993
9.0	000	001	006	021	055	116	207	324	456	587	706	803	876	926	959	978	989
9.5	000	001	004	015	040	089	165	269	392	522	645	752	836	898	940	967	982
10.0	000	000	003	010	029	067	130	220	333	458	583	697	792	864	917	951	973
10.5	000	000	002	007	021	050	102	179	279	397	521	639	742	825	888	932	960
11.0	000	000	001	005	015	038	079	143	232	241	460	579	689	781	854	907	944
11.5	000	000	001	003	011	028	060	114	191	289	402	520	633	733	815	878	924
12.0	000	000	001	002	008	020	046	090	155	242	347	462	576	682	772	844	899
12.5	000	000	000	002	005	015	035	070	125	201	297	406	519	628	725	806	868
13.0	000	000	000	001	004	011	026	054	100	166	252	353	463	573	675	764	835
13.5	000	000	000	001	003	008	019	041	079	135	211	304	409	518	623	718	798
14.0	000	000	000	000	002	006	014	032	062	109	176	260	358	464	570	669	756
14.5	000	000	000	000	001	004	010	024	048	088	145	220	311	413	518	619	711
15.0	000	000	000	000	001	003	008	018	037	070	118	185	268	363	466	568	664
16	000	000	000	000	000	001	004	010	022	043	077	127	193	275	368	467	566
17	000	000	000	000	000	001	002	005	013	026	049	085	135	201	281	371	468
18	000	000	000	000	000	000	001	003	007	015	030	055	092	143	208	287	375
19	000	000	000	000	000	000	001	002	004	009	018	035	061	098	150	215	292
20	000	000	000	000	000	000	000	001	002	005	011	021	039	066	105	157	221

TABLE 6 (Continued)

	c—Acceptable Number of Defects															
np'	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
6.6	1,000															
6.8	1,000															
7.0	1,000															
7.2	999	1,000														
7.4	999	1,000														
7.6	999	1,000														
7.8	999	1,000														
8.0	998	999	1,000													
8.5	997	999	999	1,000												
9.0	995	998	999	1,000												
9.5	991	996	998	999	1,000											
10.0	986	993	997	998	999	1,000										
10.5	978	988	994	997	999	999	1,000									
11.0	968	982	991	995	998	999	1,000									
11.5	954	974	986	992	996	998	999	1,000								
12.0	937	963	979	988	994	997	999	999	1,000							
12.5	916	948	969	983	991	995	998	999	999	1,000						
13.0	890	930	957	975	986	992	996	998	999	1,000						
13.5	861	908	942	965	980	989	994	997	998	999	1,000					
14.0	827	883	923	952	971	983	991	995	997	999	999	1,000				
14.5	790	853	901	936	960	976	986	992	996	998	999	999	1,000			
15.0	749	819	875	917	947	967	981	989	994	997	998	999	1,000			
16	659	742	812	868	911	942	963	978	987	993	996	998	999	999	1,000	
17	564	655	736	805	861	905	937	959	975	985	991	995	997	999	999	1,000
18	469	562	651	731	799	855	899	932	955	972	983	990	994	997	998	999
19	378	469	561	647	725	793	849	893	927	951	969	980	988	993	996	
20	297	381	470	559	644	721	787	843	888	922	948	966	978	987	992	995

np' = number of samples (*n*) multiplied by the defect fraction *p'*.

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TABLE 7 Factors for computing control chartlines

Observations in Sample, n	Chart for Averages			Chart for Standard Deviations						Chart for Ranges						
	Factors for Control Limits			Factors for Central Line		Factors for Control Limits				Factors for Central Line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653

TABLE 7 (Continued)

16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.135	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541
Over 25	$3/\sqrt{n}$	...	...	...	...	...[a]	...[b]	...[a]	...[b]	...	...	...	...	...	...	...

SOURCE: Reprinted courtesy of the American Society for Testing and Materials, Philadelphia, Pa.